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THE

PHYSICAL REVIEW.

SOME NEW DIFFRACTION PHOTOGRAPHS.

By Mason E. Hufford.

IT is well known that if monochromatic light from a point source is made to pass through a circular opening and fall upon a screen and if the size of the opening is chosen so that the difference in optical paths by way of the boundary and by way of the center of the opening is some whole number of half wave-lengths of the light used, then in the illuminated area where the paths end there will be interference. At the center of the area the interference will be constructive or destructive depending upon whether the path difference is an odd or an even number of half wave-lengths of the light. Openings of this sort may be considered as being made up of a series of annular rings or half period elements—due to a point source of light at a finite distance—similar to the Huyghens half period elements in the case of a plane wave. When the number of half period elements is even they destroy the effect of one another in pairs and the center of the area is black. When the number is odd the effect of one ring is not destroyed and the center is white.

Photograph *b* shows how this theory is verified by a photograph of the diffraction patterns from a series of openings varying consecutively from one to twenty-five half period elements in size. The light chosen to obtain Photograph *b* was the intense fluting beginning at wave-length $3{,}880 \times 10^{-8}$ cm. in the spectrum of the carbon arc. This region appeared most effective on the photographic plate used. It was isolated by means of a spectrometer as shown in Fig. 1.

Light from an arc *A* was focused on the slit of a spectrometer *Sp* from which the eyepiece was removed. The desired spectral region was focused on an opening .3 mm. in diameter through which the light passed along the axis of a light tight box 12.3 meters long and 20 cm. by 20 cm. in cross section. At the center of the length of the box and at right angles to the light rays was placed a brass plate .8 mm. thick through which had been drilled twenty-five circular holes ranging from 2.185 mm. to 10.927 mm. in diameter. Since it is difficult to drill accurately circular openings in a plate so thin, the plate was first bolted securely at the corners and

soldered at the edges to another plate of equal thickness. Thus burring of the drills was prevented. The desired diameters were indicated on long accurately gauged tapering reamers and these used to finish the drilling. The diameters of the holes were measured by means of a dividing engine. It is probable that the calculated and actual diameters did not differ by more than .0005 cm. Photograph *a*, reduced to one half,

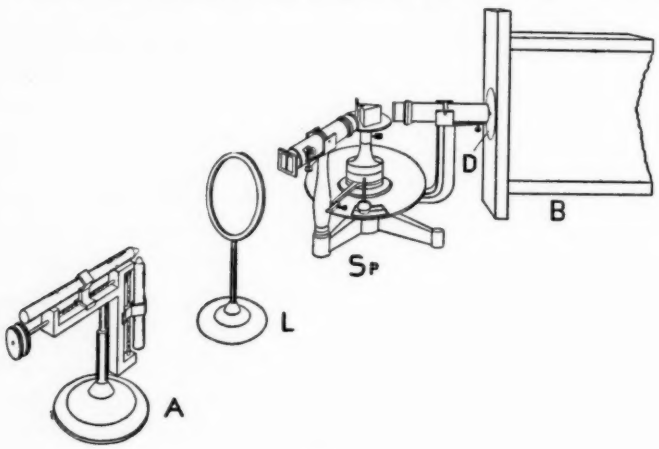


Fig. 1.

shows the arrangement of these holes. Photograph *b* (actual size) shows the resultant diffraction patterns. The alternate light and dark centers show clearly the constructive and destructive interference depending upon an odd or even number of half period elements in the openings.

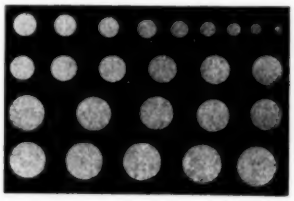
In order to photograph the Arago white spot at the center of the geometric shadow of large discs and spheres, apparatus was arranged as shown in Fig. 2.

Here *S* represents the disc or sphere suspended by fine wires in the box previously mentioned. Reflection from the walls was prevented by diaphragms *Sc*. The discs ranged from 2.55 cm. to 6 cm. in diameter and the spheres from .382 cm. to 6.35 cm. in diameter. The bright spot was obtained using any one of the discs or spheres and varied in diameter from .3 mm. to 1 cm. depending upon the size of the aperture which admitted light to the box.

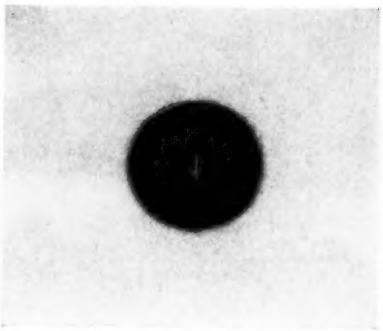
The very large bright spots thus obtained show that each point in the aperture produces a corresponding effect in the shadow. This suggests therefore, that the aperture may be any sort of figure and in the shadow of the disc or sphere there should be an inverted image of it.

To test this conclusion an aperture 1.4 cm. high by .8 cm. wide, of the form of a monogram of the letters *I* and *U* was made in a thin aluminium

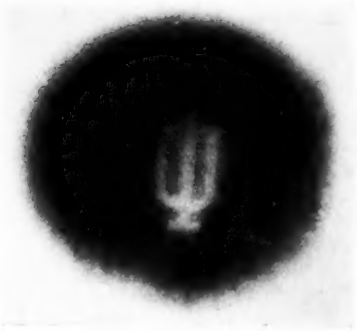
e. This aperture was mounted over the opening in the box and the



a



c

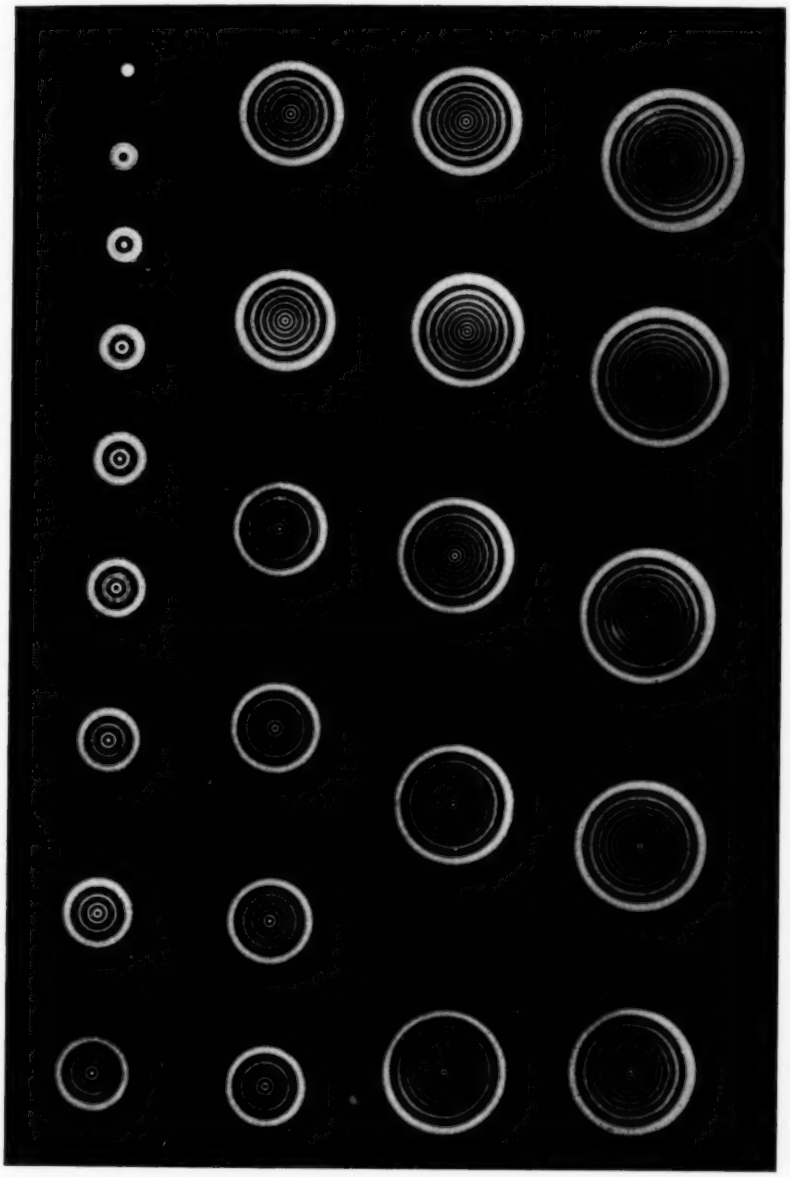


d

e

f

MASON E. HUFFORD.



b

MASON E. HUFFORD.

arc focused upon it as shown in Fig. 2. Photographs *c* and *d* show the diffraction images obtained.

It is easily seen that moving the disc or sphere nearer the aperture produces an enlarged image while moving it farther away produces a smaller one. For Photograph *c* (reduced to two-thirds) the sphere was 3.5 cm. in diameter, the distance from the sphere to aperture was 8.2 meters and from sphere to photographic plate was 4.1 meters. For

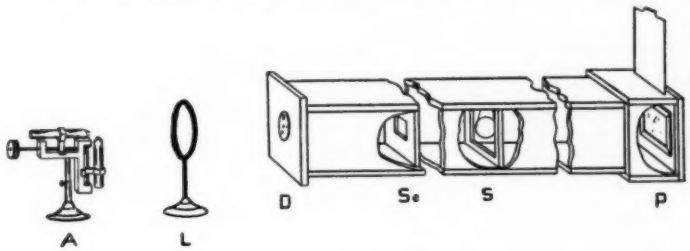


Fig. 2.

Photograph *d* (reduced to two-thirds) using the same sphere, the distances from sphere to aperture and from sphere to photographic plate were reversed.

From these photographs it is seen that with sufficient contrast on a photographic transparency an actual picture may be made, in which a disc or sphere replaces a lens.

Photograph *e* (actual size) represents a transparency 3.4 cm. high by 2.7 cm. wide which was used for this purpose. In this case the box was 23 meters long and 35 cm. by 35 cm. in cross section. The sphere was suspended 13.5 meters from the aperture over which the transparency was mounted. Photograph *f* (actual size) was obtained by using the transparency described above and a steel sphere 4.5 cm. in diameter as a diffraction object. The positive carbon of the arc was focused on the aperture (in this case the transparency) and the photographic plate exposed for ten seconds.

All of the results here described are in complete accord with diffraction theory. Photographs *c*, *d* and *f* show that by using a disc or sphere, diffraction images may be obtained which in size and position are analogous to those obtained from lenses and with a definition that is surprisingly good. In none of the photographs shown was the negative intensified or retouched.

It gives me great pleasure to acknowledge here my indebtedness to Professor A. L. Foley, whose suggestions and interest in this work have been most helpful.

LABORATORY OF PHYSICS, INDIANA UNIVERSITY,
December, 1913.

THE PRINCIPLE OF SIMILITUDE.

BY RICHARD C. TOLMAN.

THE purpose of the following article is to present some considerations which appear to have validity throughout the field of physical science. Our conclusions will all be drawn from a single postulate which we shall call the principle of similitude. This new principle may be stated as follows: *The fundamental entities out of which the physical universe is constructed are of such a nature that from them a miniature universe could be constructed exactly similar in every respect to the present universe.*

For particular kinds of dynamical systems a somewhat similar hypothesis was advanced by Newton[1] but we shall see that any complete development of the consequences of our postulate is dependent both on a knowledge of the electron theory and the theory of relativity.

We shall find that our principle provides a very simple and general method for obtaining conclusions as to the form of functional relations connecting physical magnitudes. As examples of the method we shall first deduce a number of relations in various branches of physics which will be found in agreement with those that can be obtained by more specific methods of attack. We shall then use the principle for obtaining a conclusion as to the nature of gravitational action. In a later article we shall use the principle of similitude in deriving a formula for the specific beat of homogeneous isotropic elastic solids. In the future we may regard the principle of similitude at least as a temporary criterion for the correctness of physical theories which may be advanced.

In order to derive the desired conclusions from our postulate let us consider two observers, O and O', provided with instruments for making physical measurements. O is provided with ordinary meter sticks, clocks and other measuring apparatus of the kind and size which we now possess, and makes measurements in our present physical universe. O', however, is provided with a shorter meter stick, and correspondingly altered clocks and other apparatus so that he could make measurements in the miniature universe of which we have spoken, and in accordance with our postulate obtain exactly the same numerical results in all his experiments as does O in the analogous measurements made in the real universe.

[1] Principia, lib. II., prop. 32.

Units of Length.

Let the meter stick used by O' be shorter than that used by O in the ratio $1 : x$. Hence if O and O' should both measure the *same* given distance they would find for it respectively the lengths l and l' connected by the relation

$$l' = xl. \tag{1}$$

Units of Time.

We may next inquire as to the units of time employed by the two observers. Since by our postulate we could construct for O' a universe which would appear to him exactly the same as the actual universe does to O, it is obvious that the velocity of light in free space must measure the same both for O and O'. The *only way*[1] in which this can be possible is for O' to use a shorter unit of time than does O, shorter in the ratio of $1 : x$. This shorter unit of time will then exactly compensate for the shorter meter stick, and O' will thus obtain the same numerical value for the velocity of light as does O. If now O and O' should both measure the *same* interval of time they would find for it respectively the number of seconds t and t' connected by the relation

$$t' = xt. \tag{2}$$

Units of Velocity and Acceleration.

Since we have seen that O' uses units of length and time both of which are shorter than those of O in the ratio $1 : x$, it is evident that any *given* velocity will appear the same both to O and to O' giving us the relation,

$$v' = v. \tag{3}$$

Furthermore, since acceleration has the dimensions $[l]\,[t]^{-2}$ it is evident that their measurements of a *given* acceleration will be connected by the relation

$$a' = \frac{(1)}{x} a. \tag{4}$$

Units of Electricity.

We must next inquire as to the relation between measurements of a given electrical charge as made by the two observers. In accordance with the electron theory we may accept the general principle that electricity is not a continuum but exists in definite amounts, each elementary charge being that of the electron. From this fundamental point of view

[1] The possibility of making the velocity of light appear the same to O' as to O by filling O''s space with an ether having different properties from that of O is excluded, since in accordance with the theory of relativity free space contains no ether.

the proper way to measure an electrical charge is to count the number of electrons which it contains, the fundamental unit of electricity will be the charge of the electron, and the magnitude of a charge will be expressed as an integral number. Now it is evident that if O and O' should examine the *same* electrical charge and count the number of electrons which it contains they would necessarily arrive at the same result, and hence if e and e' are their values for the magnitude of a *given* charge, we shall have the relation

$$e' = e. \tag{5}$$

Units of Mass.

In order to obtain a relation between the units of mass employed by O and O' we may consider how a simple electrostatic experiment would appear to the two observers. Consider two charges of electricity e_1 and e_2 placed on suitable bodies and separated by a considerable distance l. From Coulomb's law the force between the charges is e_1e_2/l^2, and hence if we release one of the bodies which has the mass m it will obtain the acceleration a as given by the following equation:

$$ma = \frac{e_1e_2}{l^2}. \tag{6}$$

We may suppose the quantities in this equation to have been measured by observer O. If O', however, should also observe the same experiment, it is evident from the principle of similitude that he too would have to find Coulomb's law obeyed and would obtain the relation

$$m'a' = \frac{e_1'e_2'}{l'^2}. \tag{7}$$

We have, however, transformation equations for all the quantities in this equation except m'. These equations, Nos. (1), (4) and (5), give us on substitution the relation

$$m'\frac{a}{x} = \frac{e_1e_2}{x^2l^2}. \tag{8}$$

Combining with equation (6) we obtain the desired relation between the measurements of a given mass as made by the two observers, namely:

$$m' = \frac{(1)}{x}m. \tag{9}$$

Further Transformation Equations.

We have now obtained transformation equations for the fundamental magnitudes, length, time and mass, and can hence obtain a whole series

of further equations for force, temperature, etc., by merely considering the dimensions of the quantity in question.

Since force has the dimensions $[m]\,[l]\,[t]^{-2}$ we shall obtain the transformation equation

$$f' = \frac{(1)}{x^2} f. \tag{10}$$

Energy and *absolute* temperature both have the dimensions of $[m]\,[l]^2\,[t]^{-2}$, leading to the transformation equations

$$E' = \frac{(1)}{x} E \tag{11}$$

and

$$T' = \frac{(1)}{x} T. \tag{12}$$

It should be pointed out that the transformation equation for energy has the same form as for mass, which agrees with the requirements of the theory of relativity, which has made mass and energy identical.

For area we shall evidently obtain the transformation equation,

$$S' = x^2 S. \tag{13}$$

For volume,

$$V' = x^3 V. \tag{14}$$

For pressure,

$$P' = \frac{(1)}{x^4} P. \tag{15}$$

For density of mass or energy,

$$u' = \frac{(1)}{x^4} u. \tag{16}$$

For frequency,

$$\nu' = \frac{(1)}{x} \nu. \tag{17}$$

Determination of the Form of Functional Relations from the Theory of Similitude.

Having obtained the above transformation equations for physical measurements made by the two observers O and O', we may make use of them for finding the necessary form of a number of relations between physical magnitudes. Our general method of procedure will be to consider some construct which could exist either in the actual universe or in the miniature universe which to observer O' appears the same as the actual universe. It is evident from the principal of similitude that the properties of this construct will have to obey the same general laws, whether measured by observer O or by observer O', while a further con-

dition will be imposed upon the magnitude of these measurable properties by the transformation equations which we have just developed. These two sets of conditions will permit the attainment of definite information as to the necessary form of the functional relation connecting the measurements of different properties of the construct.

THE PROPERTIES OF AN IDEAL GAS.

Let us first consider an ideal gas made up of rigid, elastic, material particles. It is obvious that such a construct would appear to be an ideal gas both to observer O and to observer O', although in accordance with equation (9) the mass of each particle would appear to O' to be $m' = \frac{(1)}{x} m$, where m is the mass as it appears to O.

The Law of Charles.—Suppose now we are interested in the way in which the pressure volume product of such a gas would vary with the temperature, we have,

$$PV = F(T), \tag{18}$$

where $F(T)$ is the unknown function whose form we wish to determine. If there is a general law connecting the pressure volume product and the temperature of an ideal gas, it is evident from the principle of similitude that this law must also apply to measurements made by observer O', and hence we shall also have

$$P'V' = F(T'),$$

where F is the same function as in the previous equation. Substituting for the accented letters their values as given by transformation equations, Nos. (15), (14) and (12), we obtain

$$\frac{PV}{x} = F\left(\frac{T}{x}\right)$$

and combining with equation (18) we have

$$PV = xF\left(\frac{T}{x}\right) = F(T).$$

Since x may be any number the only solution of this functional equation is $F(T) = kT$, where k is some constant which leads to the relation

$$PV = kT.$$

In other words, we have derived from the principle of similitude the law of Charles for an ideal gas.

The Specific Heat of an Ideal Gas.—Let us suppose that the energy of

such an ideal gas is dependent merely on the temperature. We have,

$$E = F(T) \tag{19}$$

and from the principle of similitude

$$E' = F(T').$$

Substituting transformation equations (11) and (12) we have,

$$\frac{E}{x} = F\left(\frac{T}{x}\right)$$

and combining with (19) we have,

$$E = xF\left(\frac{T}{x}\right) = F(T),$$

a functional equation for which the only solution is,—

$$E = kT,$$

where k is some constant.

This proves that the energy content of such an ideal gas is proportional to its temperature, or that the specific heat is independent of the temperature, a relation which is known to hold for gases which can be considered as composed of elastic, rigid, material particles.

THE PROPERTIES OF THE HOHLRAUM.

A hohlraum is another construct which would obviously appear as such both to observer O and observer O', although in accordance with our transformation equations its temperature and the frequency of the radiation which it contains would appear different to the two observers.

The Energy Density in Thermodynamic Equilibrium.—Consider for example a hohlraum which is in thermodynamic equilibrium; we may determine the law connecting the energy density and the temperature. We have,

$$u = F(T), \tag{20}$$

where F is the unknown function whose form we wish to determine. And from the principle of similitude we also have,

$$u' = F(T').$$

But from the transformation equations (16) and (12) we obtain,

$$\frac{u}{x^4} = F\left(\frac{T}{x}\right).$$

Combining with (21) we have,

$$u = x^4F\left(\frac{T}{x}\right) = F(T),$$

and the only solution for this functional equation is

$$u = aT^4,$$

where a is some constant. Hence we see that the principle of similitude has led to Stefan's law for the energy density of a hohlraum.

Distribution of Radiation.—It is also possible to obtain from the principle of similitude some information concerning the distribution of energy among the different wave-lengths. Let $u_\nu = \frac{du}{d\nu}$ be the rate of change of of the energy density with the frequency.

We have,

$$u_\nu = \frac{du}{d\nu} = F(\nu, T) \tag{21}$$

and from the principle of similitude,

$$u' = \frac{du'}{d\nu'} = F(\nu', T').$$

Substituting transformation equations (16), (17) and (12) we obtain,

$$\frac{u_\nu}{x^3} = F\left(\frac{\nu}{x}, \frac{T}{x}\right),$$

$$u_{\nu} = F(\nu, T) = x^3 F\left(\frac{\nu}{x}, \frac{T}{x}\right).$$

Unfortunately, this functional equation has no unique solution; it is important to point out, however, that a particular solution of our equation is the functional relation

$$u_\nu = F(\nu, T) = \nu^3 F^1\left(\frac{T}{\nu}\right)$$

which Wien has shown to be a necessary condition for any radiation equation.

THE PROPERTIES OF THE ELECTROMAGNETIC FIELD.

The principle of similitude leads to simple proofs of a number of important relations in the theory of electromagnetism.

Energy Density of an Electrostatic Field.—Suppose, for example, we wish to determine how the density of the energy u in an electrostatic field depends upon the field strength E. We have,

$$u = F(E), \tag{22}$$

and from the principle of similitude,

$$u' = F(E').$$

Now the field strength E has the dimensions of force per unit char e so that by applying transformation equations (16), (10) and (5) we obtain

$$\frac{(1)}{x^4}u = F\left(\frac{E}{x^2}\right),$$

and by combining with equation (22) we have,

$$u = F(E) = x^4 F\left(\frac{E}{x^2}\right),$$

for which the only solution is

$$u = kE^2,$$

where k is a constant. In other words, the energy density of an electrostatic field is proportional to the square of the field strength. By similar methods we could show that the energy density of a magnetic field is proportional to the square of the magnetic field strength.

There are many other electromagnetic relations upon which light is thrown by the theory of similitude. We shall content ourselves, however, by pointing out that the five fundamental equations of electromagnetic theory

$$\text{Curl } \mathbf{H} = 4\pi\mathbf{k} + \frac{(1)}{c}\frac{d\mathbf{E}}{dt},$$

$$\text{Curl } \mathbf{E} = -\frac{(1)}{c}\frac{d\mathbf{H}}{dt},$$

$$\text{div } \mathbf{E} = 4\pi\sigma,$$

$$\text{div } \mathbf{H} = O,$$

$$\mathbf{F} = \mathbf{E} + \frac{(1)}{c}\mathbf{v} \times \mathbf{H},$$

are in complete accord with the principle of similitude as will be seen by the application of the transformation equations which we have presented.[1]

PROPERTIES OF THE ELECTRON.

The principle of similitude permits us to draw two interesting conclusions as to the properties of the electron. We may consider an electron as a sphere of radius r containing the unit quantum of electricity.

Relation between Mass and Radius of an Electron.—We may now deter-

[1] For example, consider the first of these equations, curl $H = 4\pi K + (1/c)(dE/dt)$. If the principle of similitude is correct we must also have as a true equation, curl $H' = 4\pi K' + (1/c')(dE'/dt')$. Now the curl operation is essentially a differentiation with respect to length, and the transformation equation for magnetic field strength is the same as for force, so that we could put curl $H' = (1/x)$ curl $(H/x^2) = (1/x^3)$ curl H. For current density our transformation equations would evidently give us $k' = (1/x^3)k$, and for $(1/c')(dE'/dt')$ we can evidently write $(1/x^3c)(dE/dt)$. Substituting above, we obtain $(1/x^3)$ curl $H = (4\pi/x^3)k + (1/x^3c)(dE/dt)$, but this equation evidently reduces to the one we started with, thus showing no conflict with the requirements of the theory of similitude.

mine how the mass of an electron would depend on its radius. We have,

$$m = F(r),$$

and from the principle of similitude,

$$m' = F(r'). \tag{23}$$

Applying transformation equations (9) and (1) we have,

$$\frac{m}{x} = F(xr).$$

Combining with (23) we obtain

$$m = xF(xr) = F(r),$$

for which the only solution is

$$m = \frac{k}{r},$$

where k is a constant. Hence, according to the principle of similitude, the mass of the electron would be inversely proportional to the radius, a relation which can also be obtained by the more elaborate calculations of electromagnetic theory.

Radiation from an Electron.—We may also determine with considerable ease the relation between the energy radiation from an electron and its acceleration. We have

$$\frac{dE}{dt} = F(a) \tag{24}$$

and from the principle of similitude,

$$\frac{dE'}{dt'} = F(a').$$

Substituting transformation equations (11), (2) and (4) we have

$$\frac{dE}{x^2 dt} = F(a),$$

and combining with (24) we obtain

$$\frac{dE}{dt} = x^2 F\left(\frac{a}{x}\right) = F(a),$$

for which the only solution is

$$\frac{dE}{dt} = ka^2,$$

where k is a constant. We thus see that the rate of emission of energy from an accelerated electron is proportional to the square of the acceleration.

The Theory of Gravitation.

In what has preceded we have shown that the principle of similitude provides a simple method for obtaining relations in the most diverse fields of physical science. These relations can all be obtained, however, by the more specific methods of attack used in the particular branches of science under consideration. We shall now point out that in the field of gravitation theory an acceptance of the principle of similitude will lead to quite new conclusions.

Science has long been troubled by questions as to the mechanism by which gravitational forces are produced. On the one hand, almost countless hypotheses have been advanced to explain gravitation by the action of moving corpuscles, ether waves, or some electromagnetic influence, while, on the other hand, it has been warmly urged that gravitational attraction is an inherent property of the mass of a body, and that, having found in Newton's law an exact description of the way in which this attraction acts, any search for a mechanism by which the force is produced is meaningless. An acceptance of the principle of similitude, however, will force us to believe that the gravitational attraction between two bodies is not merely a function of the masses of the bodies and the distances between them, but must depend on something else as well, perhaps, for example, on the properties of some intervening medium.

To prove our point let us assume that the gravitational attraction between two bodies does depend *merely* on their masses m_1 and m_2, and the distance l between them. We have from Newton's law

$$f = k\frac{m_1 m_2}{l^2}. \tag{25}$$

But if our assumption that gravitation depends *merely* on the physical entities m_1, m_2 and l is correct, it is evident from the principle of similitude that we must have for the same system

$$f' = k\frac{m_1' m_2'}{l'^2}.$$

But substituting transformation equations (10), (9) and (1) we obtain

$$\frac{f}{x^2} = k\frac{\frac{m_1}{x}\frac{m_2}{x}}{x^2 l^2},$$

$$f = \frac{l}{x^2}\, k\frac{m_1 m_2}{l^2},$$

an equation which does not agree with the one with which we originally started, No. (25).

Of course this absurd conclusion might merely mean that the principle of similitude is not universally true. If, however, we have accepted the principle, there are two possible solutions of the problem. In the first place gravitational action may really be proportional not to mass but to some quantity which is itself more or less accidentally proportional to mass, and which like electrical charge appears of the same magnitude both to observer O and to observer O'. A second possible solution of the problem is that the attraction of gravitation does not depend merely on the masses of the attracting bodies and the distance between them, but also on the properties of some mechanism by which gravitational action is produced. The search for the true nature of gravitational action will now become an important problem of physics, and the principle of similitude will be a criterion for judging the correctness of proposed solutions.

Let us suppose, for example, that the force of gravitation depends not only on the masses of the gravitating bodies and the distance between them, but on the magnitudes, A, B, C, etc., of some properties of a gravitational mechanism. We shall then have,

$$f = F(A, B, C \cdots) \frac{m_1 m_2}{l^2},$$

and from the principle of similitude,

$$f' = F(A', B', C' \cdots) \frac{m_1' m_2'}{l'^2}.$$

Let us assume that the transformation equations for A, B, C, etc., are of the form $A' = x^a A$, $B' = x^b B$, etc., we may then obtain

$$f = F(A, B, C \cdots) \frac{m_1 m_2}{l^2} = F(x^a A, x^b B, x^c C \cdots) \frac{m_1 m_2}{x^2 l^2}$$

as an equation which must be fulfilled by a successful hypothesis for the explanation of gravitational attraction.[1]

CONCLUSION.

In the preceding article we have seen that the principle of similitude can be used for the derivation of a large number of physical relations. The methods to be applied have the advantage of great simplicity and generality, but the disadvantage of not providing any information as to the magnitude of the numerical constants which enter the equations.

[1] The writer first conceived the idea of the principle of the relativity of size eight or nine years ago, and for more than a year has been engaged in a definite attempt to draw useful conclusions from its corollary the principle of similitude. His progress has always been stopped, however, by the apparent failure of the phenomena of gravitation to meet the criterion of similitude. It is now hoped that the above treatment of the gravitational problem successfully removes this difficulty.

We have also seen that the principle of similitude could be of use for testing new physical theories which may be advanced.

In conclusion we may point out that our fundamental postulate is, as a matter of fact, a relativity principle. Indeed it might be called the principle of the relativity of size. Our postulate states that the fundamental physical entities are of such a nature that from them a miniature universe could be constructed exactly similar in every respect to the present universe, and in the transformation equations which we have developed we have shown just what changes would have to be made in lengths, masses, time intervals, energy quantities, etc., in order to construct such a miniature world. If, now, throughout the universe a simultaneous change in all physical magnitudes of just the nature required by these transformation equations should suddenly occur, it is evident that to any observer the universe would appear entirely unchanged. The length of any physical object would still appear to him the same as before, since his meter sticks would all be changed in the same ratio as the dimensions of the object, and similar considerations would apply to intervals of time, etc. From this point of view we see that it is meaningless to speak of the absolute length of an object, all we can talk about are the relative lengths of objects, the relative duration of intervals of time, etc., etc. The principle of similitude is thus identical with the principle of the relativity of size.

BERKELEY, CAL.,
January 18, 1914.

CERTAIN EXPERIMENTS IN SOUND DIFFRACTION.

BY G. W. STEWART AND HAROLD STILES.

THE general character of sound diffraction is well understood and there is little likelihood that experimental study in this field will lead to important discoveries. Nevertheless such experimental examination is highly desirable. The selection of experiments recorded in this paper was influenced by apparatus already constructed and immediately available.

THE SHADOW OF A RIGID SPHERE.

The theoretical investigation of the acoustic shadow of a rigid sphere was first made by Rayleigh[1] and was later extended by one of the writers[2] to the more general case where the distance from the sphere concerned is not limited. This theory was tested by the writers[3] and it was found that the experimental results varied from theoretical prediction in a manner indicating errors due to the distortional effect of resonance.

Apparatus.—In those experiments the source of sound was located on a hollow cement sphere, circumference 135.9 cm. This sphere was mounted near the edge of the roof of the physics building and the sound issued from it in a 5 cm. pipe opening. The sound-measuring device was a modified form of Rayleigh disc described in a previous article.[4] The sphere was rotated about a vertical axis which permitted the disc to remain stationary. The sound was produced by an electromagnetically operated tuning fork, 256 d. v., mounted on a resonator box which was introduced into a funnel from which the sound was conducted 7 meters to the vertical pipe entering the sphere.

The same sphere was used in the experiments here recorded but the disc was removed to the floor below and a 5 cm. pipe led from the disc to the position desired in the neighborhood of the sphere. The constancy of the source of sound was all that could be desired. The contact terminal attached to the fork was a short helical spring, 0.3 cm. in radius, of platinum wire, No. 29 B. and S. gauge. When the fork was actuated by storage battery voltage, it would operate for an hour without a change in its amplitude of one per cent.

[1] Rayleigh's Theory of Sound, 1896, Vol. II., p. 254.

[2] Stewart, PHYS. REV., Vol. XXXIII., No. 6, Dec., 1911, p. 467.

[3] Stewart and Stiles, PHYS. REV., Vol. I., No. 4, April, 1913, p. 309.

[4] See Stewart and Stiles, loc. cit.

As shown in Fig. 1, the sphere was rotated about a vertical axis. The Rayleigh disc was removed to the inside room below to prevent the disturbance due to air currents and to decrease the absorption due to resonance at the sphere, it having been observed that both of these errors were serious. With the pipe placed as shown in Fig. 1 the intensity produced in it was strictly proportional to the potential energies at the opening. The disc in the room below was attached directly to the pipe. Fortunately, the pipe length was such that there was no tendency for resonance. This statement is verified by the results.

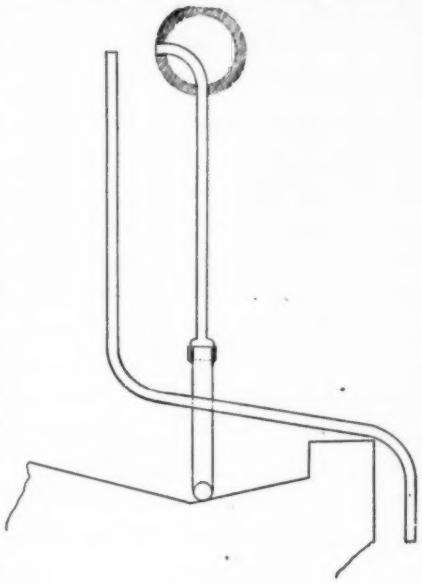


Fig. 1.

The improved apparatus demanded a more sensitive disc and this was obtained by inserting a finer quartz fiber. The period of the disc became 15 seconds.

Theory.—The mean potential energy per unit volume is given by the expression,[1]

$$\tfrac{1}{2}\rho_0(F^2 + G^2)(k/2\pi r \iint U\, dS)^2.$$

The relative intensities are therefore given by relative values of $F^2 + G^2$. The pipe opening remained at a constant distance r from the sphere center and various values of intensities were obtained by rotating the sphere, this being equivalent to a stationary sphere with the intensities measured at various points in a circumference whose plane includes the diameter of the sphere passing through the source of sound. The relative values of intensities computed as indicated in a previous article[2] for $kr = 2$, are for 0° (the position shown in Fig. 1), 1.00; 30°, 0.560; 60°, 0.187; 90°, 0.065; 120°, 0.034; 150°, 0.033; 180°, 0.033.

Results.—The results obtained for a distance from the center of the sphere equal to twice its radius are shown in Table I. This gives a record of four series of observations. The deviations from the mean can be accounted for by the smallness of the deflection, about 2 cm., and the error due to the setting of the sphere at 30° where the intensity changes

[1] For derivation and meaning of symbols see Stewart, loc. cit.

[2] For numerical values see Stewart and Stiles, loc. cit.

rapidly. In order to compare these results with the theoretical values and to show the great improvement over the earlier experimental arrangement, both the present and former results are plotted in Fig. 2, and the theoretical values are shown by the full line curve. The earlier results are represented by crosses and the later ones by small circles. The

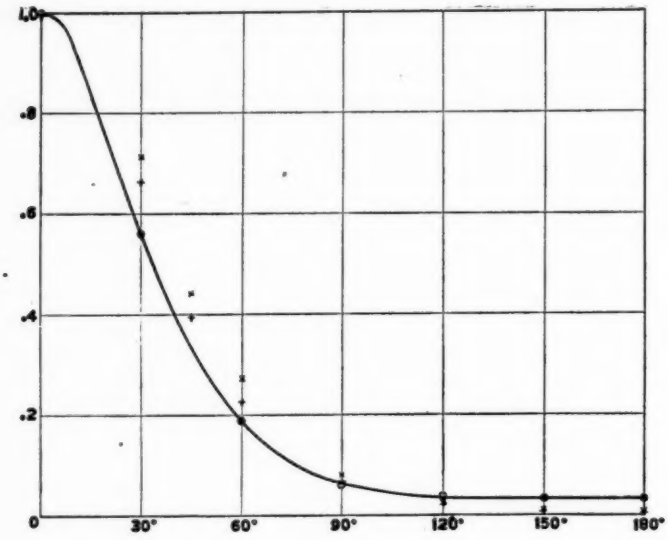


Fig. 2.

TABLE I.

	0°	30°	60°	90°	120°	150°	180°
	1.00	0.580	0.230	0.067	0.035	0.025	0.025
	1.00	0.625	0.175	0.050	0.035	0.035	0.035
	1.00	0.555	0.170	0.050	0.040	0.035	0.030
	1.00	0.475	0.185	0.075	0.045	0.040	0.035
Mean.....	1.00	0.558	0.190	0.060	0.039	0.034	0.031

agreement between theory and experiment is better than could be expected. Our belief is that the errors in the earlier results were due to air currents and to the absorption caused by resonance. The distortion produced by the latter would have the effect of "ironing out" the curve, and the former observations show that that is the case.

The verification of the theory is very satisfactory. This increases our confidence not only in the theory, but also in the Rayleigh disc. So far as we are aware, there has been no study which has shown that the deflections of the disc are proportional to the potential energy at the opening of the disc tube. It is true that the correctness of the theory

of the disc has been experimentally[1] verified so far as the relation between the kinetic energy of the air at the disc and the deflection of the disc are concerned. The writers attempted to show the proportionality of the disc deflection and the potential energy at the disc tube opening by observations of the amplitude of the fork and the deflection of the disc when both were placed in the same room. The dimensions of the room were about 25, 7, and 4 meters. The temperature as measured by wall thermometers was constant to within 1° C. It was found that, if comparisons were made quickly over the range of deflection of the disc, there was a constant proportionality between the square of the fork's amplitude and the deflection of the disc. But the apparent sensibility of the disc measured in this manner changed with time. In other words, the ratio of deflection to the square of the amplitude changed from moment to moment, being quite appreciable in a few minutes. The explanation seemed to be that the maxima and minima intensities shifted their locations with changes in temperature. The changes, however, seemed greater than could be produced by standing waves caused by only one reflection. This might have been anticipated. These tests satisfied our minds of the correctness of the assumption of proportionality of deflection and intensity at the tube opening.

PASSAGE OF SOUND THROUGH NARROW SLITS.

Rayleigh has investigated the passage of sound through narrow slits both theoretically[2] and experimentally.[3] The result of the former can be briefly presented. Consider an aperture in a thin plane screen of infinite extent. Let a plane wave be incident from the left. Its velocity potential, omitting the harmonic time factor and considering the modulus unity, is $\varphi = e^{-ikx}$. Consider the conditions without aperture and then the supplementary values of the velocity potential representing the changes produced by the aperture. By addition, the velocity potential on each side of the screen is obtained. The resulting velocity potential at a great distance, r, on the right side is,

$$\varphi = e^{-ikr}\frac{M}{r}$$

or

$$\varphi = \frac{M}{r}\cos^{\kappa(at-r)},$$

where k is $2\pi \div$ *wave-length* and where M is the "capacity." The term

[1] Zernov, Annal. d. Phys., No. 26, p. 79, 1908.

[2] Rayleigh, Phil. Mag., XLIII., p. 259 (1897); Scien. Paper IV., p. 291.

[3] Rayleigh, Phil. Mag., XIV., p. 153 (1907).

"capacity" is used because M is the total quantity of electricity which can be distributed over the aperture in a manner to produce a uniform potential of unity over the aperture. It is known that for an ellipse,

$$M = \frac{a}{F(e)},$$

where a is the major axis, and small e the eccentricity, and F is the symbol of the complete elliptic function of the first kind. If the ellipse be very elongated,

$$M = \frac{a}{\log_e\left(\frac{4a}{b}\right)}.$$

Inasmuch as the only variable in the expression for the velocity potential at a fixed great distance, r, is M, the same values of M would determine the same sound intensities. The above formula shows that the intensity is much more sensitive to alterations in a than in b, the minor axis, and Rayleigh attempted to verify this by experiment. The difficulties he encountered were great, and in addition he depended upon ear memory for the reproduction of identical intensities. The arrangement of our Rayleigh disc and the possibility of experimentation practically free from reflection, tempted us to test the above formula.

For the infinite plane a rectangular piece of galvanized iron 33 × 38 cm. was utilized, the dimensions and shape being accidental. This plate was fastened on the horizontal end of an elbow placed upon the end of the pipe shown in Fig. 1. In the center of the vertical plate an opening 0.8 × 4.0 cm. was made, and over the opening was constructed the slit. Four small safety razor blades, 1.8 × 4 cm., beveled on both sides, were used. Three were placed in the same plane and to the fourth was attached an edge made out of a copper strip, whose thickness was the desired width of the slit. Changes in the slit width required different copper strips. Changes in the length could be readily obtained by sliding the fourth blade with its attached copper strip. Vaseline was freely used to stop all openings and was found entirely satisfactory.

The source of sound was the electrically operated tuning fork with the open end of the resonator mounted directly in front of the slit and about 150 cm. distant.

Our first experiments were made with sharpened brass edges but these were found unsatisfactory when the slit became narrow. Experiments with these edges with a width of 1 mm. (b = 0.5 mm.), showed that the variation of intensity with length was practically linear. This fact simplified our experiments with the razor blades. We obtained a deflec-

tion with a 17.5 mm. and b 0.1 mm., then a deflection with a 17.5 mm. and b 0.5 mm. and finally a deflection with a 10.0 mm., and b 0.5 mm. The ratio of intensities of the first and second slits were found to be approximately 0.52, the separate values being 0.51, 0.49, 0.55. The ratio of intensities of the third and second was approximately 0.49, the separate values being 0.47, 0.50, and 0.49. By assuming that the linear relation above cited holds, it is readily seen that the value of a giving the ratio of 0.52 with b 0.5 mm., would be 10.5 mm. We would then have the same value of M for the two slits, 35. × 0.2 mm. and 22. × 1.0 mm. Substituting these experimental values, we have

$$M = \frac{a}{\log_e\left(\frac{4a}{b}\right)} = \frac{17.5}{\log_e\left(4\frac{17.5}{0.1}\right)} = 2.66$$

$$= \frac{10.5}{\log_e\left(4\frac{10.5}{0.5}\right)} = 2.37.$$

Thus the formula for M is verified as nearly as could be expected with the lack of conformity with theoretical conditions. For it is to be noted that the presence of the pipe behind the slit destroyed the plane of infinite area, and further, that there is resonance in the pipe leading to the disc. This resonance would of course be modified by the size of the aperture. Other experiments cited below would indicate that the error due to resonance is of the same order as the variation between the two values of the M given above.

Passage Through Circular Apertures.

Tests were made by varying the area and by varying the number of circular apertures of equal area. The value of M for a circle is $2a \div \pi$ where a is the radius. If the velocity potential at a great distance is proportional to M, then the intensity at a given distance, proportional to $\dot{\varphi}^2$, is proportional to M^2, or to a^2. An experimental test of this relation did not prove satisfactory, for as M changed the resonance in the pipe was altered and the conditions of the experiment were thus seriously modified. The values actually obtained are shown in the accompanying table, Table II. The deflections were reduced to the same scale, the aperture F giving a deflection of unity. The ratio in the last column is the relation of experimental and theoretical values of M. The ratio varies not more than 10 per cent. from the mean for a range of radii from 0.42 cm. to 1.28 cm. These variations, as well as those for the

smaller apertures are accounted for by variations in resonance. The indications are, however, that if resonance could be avoided a close agreement between experiment and theory would be found.

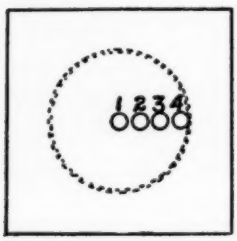


Fig. 3.

The value of M for N circles is $N \times M$, and the corresponding intensity is proportional to N^2M^2. If the M's are not alike, the intensity is proportional to the square of their sum. A test was made as follows. Four circular apertures, each 0.30 cm. in radius were made in a screen, see Fig. 3, and intensities obtained by opening them separately and simultaneously as indicated in the accompanying Table III. The agreement between theory and experiment is indicated by the ratio in the last column. It should be observed that the variation is not great and can readily be

TABLE II.

Screen.	Deflection.	$\sqrt{\text{Defl.}} \propto M$.	$2a$.	Ratio.
L	5.45	2.34	2.56	0.91
K	3.67	1.91	1.88	1.02
J	2.00	1.41	1.27	1.11
F	1.00	1.00	1.00	1.00
D	0.56	0.75	0.83	0.90
C	0.20	0.45	0.60	0.75
B	0.06	0.25	0.40	0.62

TABLE III.

Apertures Used.	Relative Observed Values of Intensity.	Relative Values of M $\sqrt{\text{Intensity}} \propto M$.	Relative Computed Values of M $M=M_1+M_2+M_3+M_4$.	Ratio.
1	1.00	1.00		
2	1.00	1.00		
3	0.80	0.90		
4	0.58	0.76		
1 + 2	3.73	1.93	2.00	0.96
1 + 2 + 3	6.23	2.50	2.90	0.86
1 + 2 + 3 + 4	9.66	3.10	3.66	0.83

interpreted as due to change of resonance in the pipe. The variations of M for the separate apertures is doubtless due to the proximity of the apertures to the pipe. The arrangement is faulty also in that the apertures are close together. The lack of agreement between the experimental and theoretical results is explained by the lack of conformity

to theoretical conditions and the variation in resonance in the pipe. The latter seems to produce the greater error. Indeed, all the experiments with apertures indicate that the errors due to changes in resonance are of the same order as the differences between experimental and theoretical results.

The results with both slits and apertures show the difficulty of securing satisfactory experimental conditions. Experiments of this character should be continued with improvements in apparatus that will avoid the errors due to resonance.

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POLARIZATION IN THE ALUMINIUM RECTIFIER.

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THE primary object of this investigation was to make a study of the potential difference between the electrodes of the aluminium rectifier at extremely short intervals after breaking the charging circuit. It was hoped that the investigation might also throw some further light on the action of electrolytic cells. The methods employed and results obtained are new and may be suggestive in the further study of the electrolytic cell.

The aluminium plates, which were used as electrodes, were prepared in the instrument shops of the University of Michigan and were hammered out of 98 per cent. pure aluminium ingots. They were 10 cm. long, 2.7 cm. wide and 0.3 cm. thick. The plates were so insulated with Bank of England sealing wax as to leave a definite area exposed as active electrode to the electrolyte. The cathode was a platinum plate of 20 sq. cm. exposed area, totally immersed in the electrolyte, the platinum wire leading out of the solution being thoroughly insulated from the solution. The insulation thus eliminated any possibility of surface effects, which are found to be present when the electrode surfaces are exposed at the surface of the electrolyte. The active surfaces of the aluminium electrodes were polished with fine sand paper and rinsed with distilled water before each experiment was begun. The electrolytes used were ammonium aluminium sulphate (40 grams per liter of water) and potassium dichromate (26.18 grams per liter of water).

After checking some of the results obtained by former investigators[1] by the ballistic galvanometer-condenser method, a potentiometer method for measuring the counter E.M.F. was devised and the results thereby obtained were compared with those obtained by the former method. The potentiometer method invariably gave larger values for the counter E.M.F. of the rectifier than did the condenser method, the differences being far beyond the range of experimental error. For instance, with a given E.M.F. the difference was consistently about 4.7 per cent. A

[1] Theory of the Electrolytic Rectifier, by S. R. Cook, PHYS. REV., 18, pp. 23–29, Jan., 1904; also PHYS. REV., 20, pp. 312–321, May, 1905; H. W. Morse and C. L. B. Shuddemagen, American Acad. Arts and Sciences, *Proc.*, 44, pp. 367–397, 1908–9.

repetition of comparative experiments gave a similar characteristic difference, which indicates that when the charged rectifier is connected to a condenser, a quantity of electricity required to charge the condenser being taken from the rectifier, the electrical condition of the rectifier is thereby disturbed and the difference of potential between the electrodes of the rectifier decreased. The potentiometer method was, therefore, used throughout the remainder of the experimental work.

The arrangement of apparatus for the measurement of the counter E.M.F. of the rectifier after the expiration of definite periods of open circuit is indicated in Fig. 1. R is the aluminium rectifier, in which A is

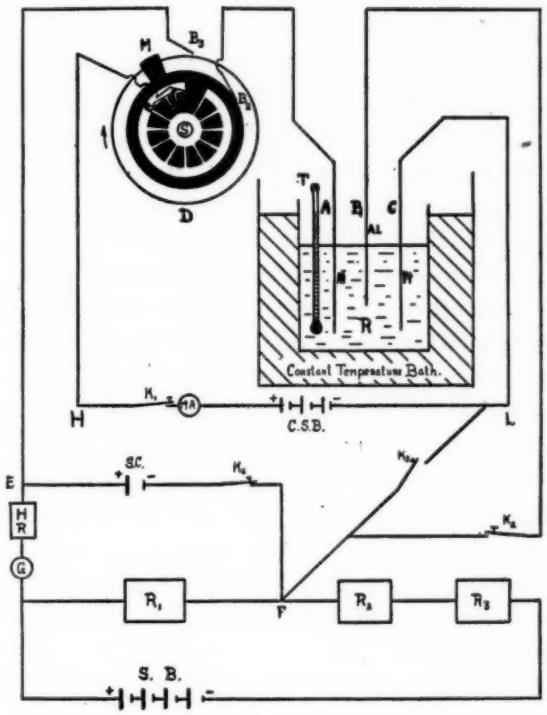

Fig. 1.

the aluminium anode; C, the platinum cathode; and B, an aluminium test electrode placed at one side and somewhat in the rear of the anode so as not to be in the path of the current between anode and cathode; *C.S.B.*, the charging storage battery; *MA*, a milliammeter; R_1 and R_2, two high grade Leeds and Northrup resistance boxes; R_3, a high grade Leeds and Northrup resistance box with travelling plug; *H.R.*, a 250,000 ohm resistance box; *S.C.* a cadmium standard cell; *G.*, a Leeds and, Northrup high sensibility galvanometer having a resistance of 1,600

ohms; *S.B.*, a storage battery; and *D*, a disc especially designed for this experimental work. The disc consisted of a single piece of hard rubber, 13.5 cm. in diameter and 1 cm. thick, to which were firmly riveted two concentric brass rings, 2 mm. thick and 1.5 cm. wide, the outside diameters being 7.5 cm. and 13 cm., respectively. The inner ring was divided into 30° sectors, one of which was subdivided into two 15° sectors; one of these 15° sectors was later soldered to the adjoining 30° sector, as indicated in the figure. These sectors could be connected, when desired, by inserting a brass plug between them. One of these sectors was electrically connected, as shown in the diagram, to the outer ring. The outer ring was in turn connected to the brass contact maker, *M*, which fitted closely over the rim of the rubber disc. The contact maker could be shifted along the rim by 15° intervals and was firmly held in place by a plug inserted through the contact maker and into the disc from the rear. The disc was rotated by a motor running at an approximately constant speed of 2,000 R.P.M. B_1, B_2, B_3 are spring brass brushes so adjusted as to make good electrical contact with the inner ring, outer ring and contact maker, respectively.

The division of the inner ring into sectors makes it possible to regulate the portion of the period of rotation during which the impressed E.M.F. is applied to the rectifier; the portion of the period finally adopted was the time required for the brush to sweep over a 45° sector. Regulation of the contact maker made it possible to vary the period of decay of the counter E.M.F. of the rectifier before this E.M.F. was balanced against the fall of potential over R_1. The sum of R_1 and R_2 was kept constant and the resistance in R_3 adjusted until the current flowing through R_1 was 0.001 ampere, as was shown, on closing K_s, by balancing the fall of potential over R_1 against the E.M.F. of the standard cell. This made it possible to read the counter E.M.F., on obtaining a balance, directly from the resistance in R_1. On closing K_1, *C.S.B.* charges the rectifier through the disc; on closing K_2 the counter E.M.F. of the rectifier is balanced against the fall of potential over R_1, while B_3 sweeps over *M*.

The arrangement of apparatus indicated in Fig. 1 is such, on closing K_2, as to give the data for the curve of decay between the aluminium anode and the aluminium test electrode, thus giving the effect at the anode only, due to the impressed E.M.F. By closing K_3 instead of K_2 the data for the curve of decay of E.M.F. between anode and cathode were obtained, thus giving the total effect of polarization. To obtain the effect of polarization at the cathode only the positive terminal of *C.S.B.* was disconnected from the disc and connected directly to the anode, while the

negative terminal of *C.S.B.* was connected to the cathode through the disc.

It was ascertained that in order to obtain the maximum counter E.M.F. of the rectifier by charging it through the disc while revolving, it was necessary that the motor run for a period of from 10 to 15 minutes before the polarization was measured and 15 minutes was the interval accordingly adopted before any measurements were taken. One of the greatest difficulties met with in experimental work with the rectifier is the slow change in the character of the film with time; and it is therefore necessary (1) that the series of results be taken as rapidly as possible, (2) that the period of open circuit between the determinations of the balance for successive intervals of decay be kept constant, (3) that the points on the decay curve be determined in the same order. A cyclic order of performing the operations was followed, thus rendering it feasible for each series of results obtained to be checked. Yet the author believes that the curves obtained are characteristic of the general behavior of the anode film.

Table I. gives the mean corrected values obtained from a number of

TABLE I.

Electrolyte: Ammonium Aluminium Sulphate.
Applied E.M.F.: 6.16 volts.
Temperature: 23° C.
Current: 0.00025 ampere.

Period of Decay, Second.	E_{A-3}, Volts.	E_{AC}, Volts.	E_{C-3}, Volts.	$E_{AC}+E_{C-3}$, Volts.	$E_{A-3}-(E_{AC}+E_{C-3})$, Volts.
0.0000	6.43	6.15	....	....	....
0.0013	6.20	5.90	0.31	6.21	−0.01
0.0025	6.02	5.66	0.36	6.02	0.00
0.0038	5.91	5.50	0.39	5.89	+0.02
0.0050	5.78	5.36	0.35	5.71	0.07
0.0075	5.57	5.17	0.36	5.53	0.04
0.0100	5.43	5.01	0.35	5.36	0.07
0.0150	5.24	4.80	0.38	5.18	0.06
0.0200	5.08	4.64	0.38	5.02	0.06
0.0250	4.90	4.51	0.40	4.91	−0.01

experiments and Fig. 2 gives the curves plotted therefrom for the decay of the counter E.M.F. between the anode and the aluminium test electrode (A–3), between the anode and cathode (AC) and between the cathode and aluminium test electrode (C–3), for an applied E.M.F. of 6.16 volts. Table II. gives the corresponding data and Fig. 3 gives the curves for an applied E.M.F. of 12.25 volts. Table III. and Fig. 4 give the corresponding results for an applied E.M.F. of 18.13 volts.

TABLE II.

Electrolyte: Ammonium Aluminium Sulphate.
Applied E.M.F.: 12.25 volts.
Temperature: 23.°2 C.
Current: 0.00035 ampere.

Period of Decay, Second.	E_{A-3}, Volts.	E_{AC}, Volts.	E_{C-3}, Volts.	$E_{AC}+E_{C-3}$, Volts.	$E_{A-3}-(E_{AC}+E_{C-3})$, Volts.
0.0000	12.52	12.31	....	.....	....
0.00083	11.92	11.72	0.31	12.03	−0.11
0.0021	11.32	11.07	0.31	11.38	−0.06
0.0033	11.00	10.69	0.31	11.00	0.00
0.0046	10.69	10.42	0.32	10.74	−0.05
0.0071	10.37	10.02	0.33	10.35	+0.02
0.0096	10.14	9.71	0.35	10.06	0.08
0.0146	9.60	9.28	0.36	9.64	−0.04
0.0196	9.37	8.89	0.37	9.26	+0.11
0.0246	9.23	8.75	0.36	9.11	0.12

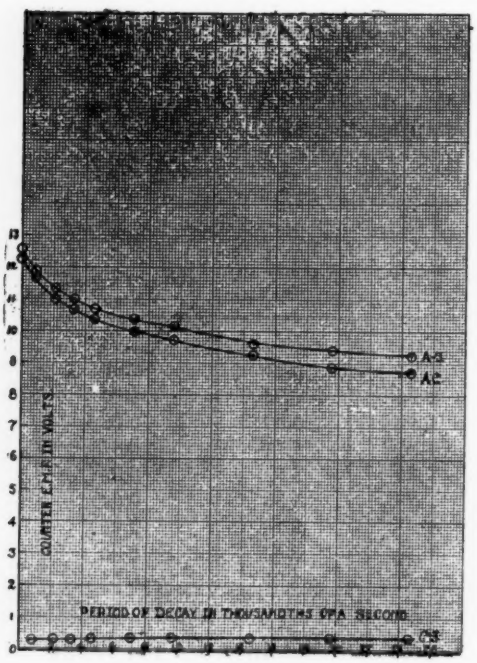

Fig. 2.

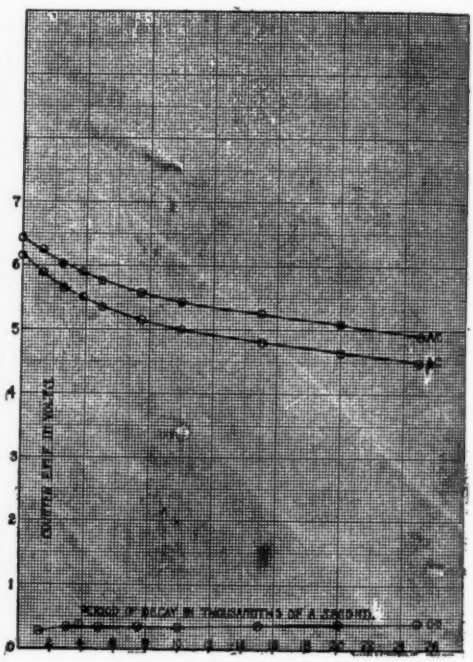

Fig. 3.

It is worthy of note that the polarization E.M.F. between the anode and the aluminium test electrode exceeds the applied E.M.F. by about 0.3 of a volt. The excess voltage is due to the fact that the electrolytic cell, acting as a primary cell, furnished an effective voltage of about 0.3

TABLE III.

Electrolyte: Ammonium Aluminium Sulphate
Applied E.M.F.: 18.13 volts.
Temperature: 23° C.
Current: 0.0005 ampere.

Period of Decay, Second.	E_{A-3}, Volts.	E_{AC}, Volts.	E_{C-3}, Volts.	$E_{AC}+E_{C-3}$, Volts.	$E_{A-3}-(E_{AC}+E_{C-3})$, Volts.
0.0000	18.43	18.02	....	....	.....
0.0013	17.01	16.59	0.34	16.93	+0.08
0.0025	16.11	15.68	0.32	16.00	0.11
0.0038	15.57	15.12	0.31	15.43	0.14
0.0050	15.29	14.70	0.31	15.01	0.28
0.0075	14.63	14.09	0.32	14.41	0.22
0.0100	14.00	13.67	0.31	13.98	0.02
0.0150	13.33	12.91	0.34	13.24	0.09
0.0200	12.98	12.53	0.36	12.89	0.09
0.0250	12.75	12.34	0.37	12.71	0.04

volt in series with the charging storage battery to assist in the process of polarization. The difference between E_{A3} and $E_{AC}+E_{C3}$ for corresponding decay intervals is also seen to be negligible, the error rarely exceeding 1 per cent., which demonstrates that the method used renders possible the attainment of a high degree of accuracy. The results show conclusively that the E.M.F. of polarization, at the instant when the charging circuit is opened, is equal to the polarizing E.M.F., and that there has been no considerable *IR* component in the surface film during the charging period.

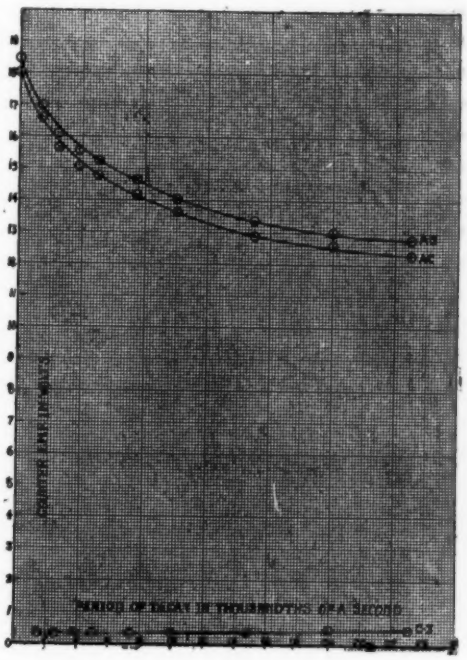

Fig. 4.

It has been proved by Schulze,[1] in an extensive series of experiments, that the effective anodic layer, in conformity with the suggestion of

[1] Ann. d. Physik, 21, 5, pp. 929–954, December 14, 1906; also 22, 3, pp. 543–558, March 5, 1907.

Guthe,[1] consists of oxygen gas. An extension of the gas film theory accounts for the slight residual current that flows through the electrolytic cell. The negative ions accumulate in the electrolyte next to the gas layer and electrons are forced from some of the carriers by the high potential gradient. These free electrons pass through the gas layer, thus causing the observed small residual current. The higher the value of the impressed voltage, under the experimental conditions maintained by the writer, the higher is the potential gradient through the gas film, and the larger should be the number of detached electrons and the resulting current. The results are seen to conform to this theory, since, for example, the residual current for an applied voltage of 6.16 volts was 0.00025 ampere; for 12.25 volts, 0.00035 ampere; for 18.13 volts, 0.00050 ampere.

The above way of looking at the counter E.M.F. of the aluminium cell very closely corresponds to our ideas of the process of charging a condenser and by many experimenters the aluminium anode is spoken of as a condenser. This comparison should, however, not be carried too far. When the two plates of a charged condenser are connected by a conductor of very high resistance, the potential difference between the plates decreases in accord with the well-known law: $V = V_0 e^{-t/RC}$. Passing to logarithms, Log V = Log $V_0 - t/RC$. If the action of the rectifier were that of an ordinary condenser, a straight line should be obtained by plotting the logarithms of V as ordinates and the corresponding periods as abscissæ. Applying this test to the data of Table IV., for instance, curve C, of Fig. 5, is obtained. It shows that the rate of decrease of counter E.M.F. is not such as to indicate that the rectifier conforms to the law of the ordinary condenser. The explana-

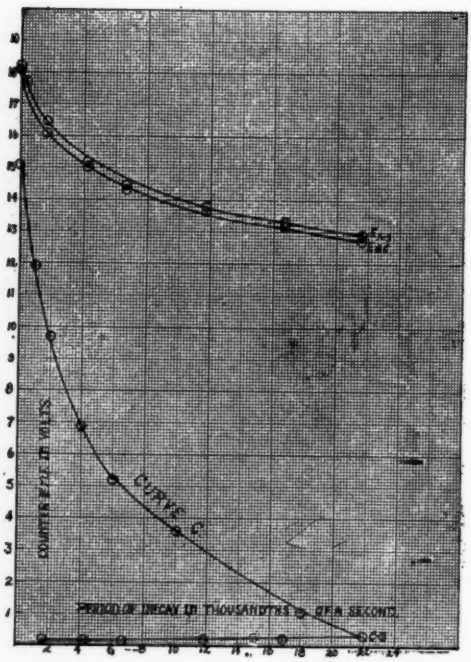

Fig. 5.

[1] Theory of Electrolytic Rectifier, PHYS. REV., 15, pp. 327–334, 1902.

tion of this discrepancy may lie in the fact that the decrease of potential in the rectifier is due to the diffusion of the ions back into the solution rather than to ordinary metallic conduction.

TABLE IV.

Electrolyte: Potassium Dichromate Solution.
Temperature: 23.1° C.
Applied E.M.F.: 18.16 volts.
Current: 0.0005 ampere.

Period of Decay, Second.	E_{A3} Volts.
0.0000	18.22
0.0004	17.70
0.0017	16.31
0.0042	15.17
0.0067	14.44

When the impressed voltage causing the polarization of the rectifier is increased the ionic concentration near the film is increased and with it the rate of change in the concentration passing from the film to the body of the electrolyte is increased. Therefore, when the impressed voltage is removed, the ions should disappear from the boundary of the gas layer the more rapidly the higher the impressed voltage. A comparison of the curves of Fig. 6 confirms this view. If the assumption that the rate of diffusion of the ions from the gas layer back into the solution is a function of the difference between the ionic concentration near the anode and that in the remainder of the electrolyte, is correct, — then as the process of diffusion proceeds and the difference in ionic concentration decreases, the rate of diffusion (or the rate of decay of the counter E.M.F. of polarization) should decrease. This is seen to be true of each of the curves of decay.

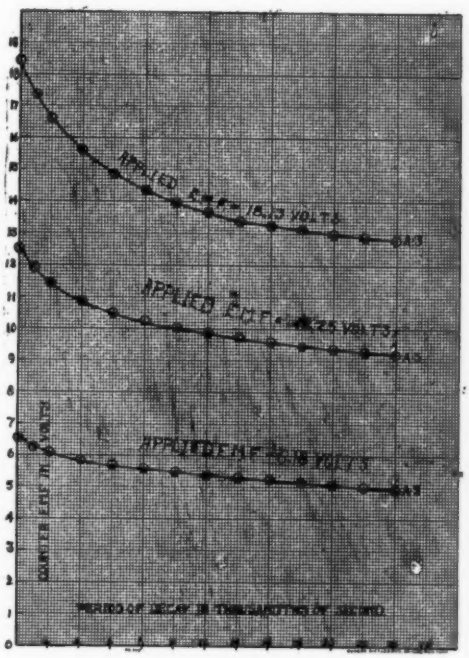

Fig. 6.

It is believed that the curves of decay obtained represent with a fair degree of accuracy no other effects than those of ionic diffusion, since during the extremely short intervals of decay used the effect of any disintegration of the film may be assumed to be negligible.

The results of this investigation may be summarized as follows:

1. The condenser method of measuring the counter E.M.F. introduces an error due to the taking of a charge from the rectifier.

2. The rectifier does not behave as an ordinary condenser.

3. When aluminium is used as the anode, the counter E.M.F. alone accounts for the reduction of the current to its exceedingly small value.

4. The results obtained in this investigation are apparently in accord with the gas film theory and the theory of ionic diffusion.

The writer is indebted to the University of Michigan for the special apparatus provided for the work and to Professor K. E. Guthe, of the University of Michigan, for many helpful suggestions received.

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A COMPLETE COLLECTION OF THERMODYNAMIC FORMULAS.

BY P. W. BRIDGMAN.

IN this paper is presented a complete and systematic collection of thermodynamic formulas involving the first and second derivatives of the ordinary thermodynamic quantities. It has been possible to compress such a collection into a reasonable compass by a short-hand method of expression. The fundamental quantities whose various relations are treated in the tables are ten in number; namely p, v, τ, E, s, Q, W, H, Z, and Ψ. It will be remarked that the specific heats are not included in the fundamental ten quantities. This is because the specific heats are properly first derivatives, $C_p = \left(\frac{\partial Q}{\partial \tau}\right)_p$, and $C_v = \left(\frac{\partial Q}{\partial \tau}\right)_v$. These ten quantities, with their first and second derivatives, are connected by various relations. The relations between the quantities themselves are simple, and mostly of the nature of definitions. But the relations between the first and second derivatives are more complicated, and it is these which are of special interest.

We consider first the first derivatives. The conditions imposed by the first and second laws of thermodynamics and the particular properties of the substance under consideration are such that every derivative of the type $\left(\frac{\partial W}{\partial \tau}\right)_Q$ has a definite meaning. This derivative means that the particular body in question is allowed to change so that Q remains constant, that is, no heat is absorbed, and the ratio of the change of W to the change of τ found during this change.[1] Every first derivative involves, therefore, three different variables, and it is at once seen that the total number of such derivatives is 720 ($= 10 \times 9 \times 8$). These 720 derivatives are connected by various relations, and in general there is an equation connecting any four of them and certain of the fundamental ten quantities. There are, therefore,

$$\frac{720 \times 719 \times 718 \times 717}{1 \times 2 \times 3 \times 4} = 11{,}104{,}365{,}420$$

such relations between the first derivatives.

[1] It should be noted that although the derivatives always have a meaning in the sense indicated, the functions which are being differentiated need not be expressible as functions of the position coördinates. W and Q are such functions; it is impossible to assign any meaning to them as functions of p and τ for example, but still the variations of W and Q in definite directions are entirely determined.

A complete collection of first derivatives would involve the tabulation, therefore, of these 11,104,365,420 relations. This of course is absolutely out of the question; the best we can do is to tabulate some of the derivatives in such a form that any of the derivatives may be obtained by slight and obvious effort. We evidently shall attain this object if every one of the 720 first derivatives is tabulated in terms of the same three fundamental derivatives. To obtain any desired one of the numerous relations between any four derivatives, therefore, we merely have to eliminate the three fundamental derivatives between the four equations for the four derivatives.

The three fundamental derivatives may be chosen in a great variety of ways. The three chosen here are the three which are perhaps given most directly by experiment; the isothermal compressibility $\left(\frac{\partial v}{\partial p}\right)_\tau$, the isopiestic dilatation $\left(\frac{\partial v}{\partial \tau}\right)_p$, and the specific heat at constant pressure C_p. The first two of these may be obtained from the characteristic equation of the substance, that is, the relation connecting p, τ, and v. To determine C_p, calorimetric measurements must be made in addition to the measurements for the characteristic equation. It may be proved that C_p is completely determined, if in addition to our knowledge of the characteristic equation, C_p is known along some line not at constant temperature. Such information, for instance, would be given by a determination of C_p as a function of temperature at atmospheric pressure.

It should be remarked that the method used here of tabulating the 720 derivatives in terms of the same fundamental three will largely do away with the necessity for determining the other relations by an elimination, as suggested above. For if in any special problem every quantity of interest is kept in terms of the same fundamental three, which are independent, one may be sure that at the end of the discussion there are no essential relations not brought to light.

It is now possible to still further reduce the number of expressions needed. To do this, the 720 derivatives may be divided into groups, the variable kept constant during differentiation being kept constant in each group. There are, therefore, ten of these groups, 72 to a group. Let us suppose, for example, that the group is that in which p is the constant element. Any one of the 72 derivatives of this group is of the type $\left(\frac{\partial x}{\partial y}\right)_p$, where x and y are any two of the nine remaining of the ten fundamental quantities. Now let us write, merely as a matter of notation, $\left(\frac{\partial x}{\partial y}\right)_p \equiv \frac{(\partial x)_p}{(\partial y)_p}$. The abridgement in the number of required formulas

is suggested by noticing that we may tabulate $(\partial x)_p$ and $(\partial y)_p$ separately as appropriate functions of $\left(\frac{\partial v}{\partial p}\right)_\tau$, $\left(\frac{\partial v}{\partial \tau}\right)_p$, and C_p, and always get the right value for the derivative by replacing $(\partial x)_p$ and $(\partial y)_p$ by these functions and taking the ratio. But there are only nine such expressions in any group, so that we have reduced the number of expressions needed from 720 to 90.

To prove the possibility of splitting up a derivative in the way above is simple. We have the mathematical identity $\left(\frac{\partial x}{\partial y}\right)_p \equiv \left(\frac{\partial x}{\partial \alpha}\right)_p \Big/ \left(\frac{\partial y}{\partial \alpha}\right)_p$ where α is any variable, not even necessarily one of the fundamental ten, which remains the same throughout the group of 72. If therefore, we replace $(\partial x)_p$ by $\left(\frac{\partial x}{\partial \alpha}\right)_p$, and similarly $(\partial y)_p$ by $\left(\frac{\partial y}{\partial \alpha}\right)_p$, we shall always obtain the right answer when we take the ratio of any two such functions to find the derivative. It is especially to be noticed that $(\partial x)_p$ is not equal to $\left(\frac{\partial x}{\partial \alpha}\right)_p$; in fact, $(\partial x)_p$ in general does not have the same dimensions as $\left(\frac{\partial x}{\partial \alpha}\right)_p$. The finite functions replacing the differentials have meaning only when the *ratio* of two is taken.

Finally it is possible to further reduce the number of expressions from 90 to 45. We notice that α, of the paragraph above, is not subject to any essential restriction—any function will do. There are ten of these α's. We may now impose a restriction, making $(\partial x)_p = -(\partial p)_x$, thus reducing to one half the number of fundamental functions. The proof of this will not be given here, but it may be readily seen on writing out the derivatives of the various groups that if an α is chosen so that the relation is satisfied for one derivative of a group, then it will be satisfied by all the others also. The α's so restricted are not completely determined by any means, but the derivatives, which only we are interested in, are now nearly determined. There is still a certain amount of arbitrariness left, in that the entire collection of functions replacing the partial differentials may be modified by the addition of any factor, but otherwise the functions are determined. This arbitrary factor will be so chosen as to make the functions as simple as possible.

In the actual derivation of the formulas, the α's play no part; they are simply the mathematical scaffolding used in proving the possibility of replacing the differentials by finite functions, and may now be completely discarded. The method actually used in deriving the formulas was to find a sufficient number of derivatives by well established methods,

and then to split them up, by inspection, into the functions replacing numerator and denominator.

We turn now to a consideration of the second derivatives. The number of combinations of second derivatives is so great that it cannot be reduced to a reasonable number, as could the number of combinations of first derivatives. All that we can do here is to tabulate a sufficient number of the fundamental second derivatives so that any relation existing between them may be found readily by such purely formal mathematical operations of differentiation or elimination as are in the mathematical equipment of every one.

The problem in the case of the second derivatives is analogous to that for the first derivatives; namely to express everything in terms of the same fundamental second derivatives. It may be shown that in general there are four such second derivatives, and for use here we choose the four most directly given by experiment, $\left(\frac{\partial^2 v}{\partial p^2}\right)_\tau$, $\frac{\partial^2 v}{\partial p \partial \tau}$, $\left(\frac{\partial^2 v}{\partial \tau^2}\right)_p$, and $\left(\frac{\partial C_p}{\partial \tau}\right)_p$.

Suppose now that we wish to find any second derivative. The general second derivative is of the type $\left[\frac{\partial}{\partial x_1}\left(\frac{\partial x_2}{\partial x_3}\right)_{x_4}\right]_{x_5}$. This is mathematically equivalent to $\left[\frac{\partial}{\partial p}\left(\frac{\partial x_2}{\partial x_3}\right)_{x_4}\right]_{x_5} \cdot \left(\frac{\partial p}{\partial x_1}\right)_{x_5}$. But $\left(\frac{\partial p}{\partial x_1}\right)_{x_5}$ is a function of $\left(\frac{\partial v}{\partial p}\right)_\tau$, $\left(\frac{\partial v}{\partial \tau}\right)_p$, and C_p, which is already known from the tables for the first derivatives. Furthermore, $\left(\frac{\partial x_2}{\partial x_3}\right)_{x_4}$ is also a known function of the same three fundamental derivatives, so that $\left[\frac{\partial}{\partial p}\left(\frac{\partial x_2}{\partial x_3}\right)_{x_4}\right]_{x_5}$ may be found by a purely formal differentiation, if we know $\left(\frac{\partial}{\partial p}\right)_{x_5}$ of $\left(\frac{\partial v}{\partial p}\right)_\tau$, $\left(\frac{\partial v}{\partial \tau}\right)_p$, and C_p. There are 27 such second derivatives. The only exception to this scheme of treatment is when $p \equiv x_5$, in which case we have an indeterminate form to evaluate. This may be avoided by taking τ instead of p as our auxiliary variable of differentiation. The differentiation in this case is so simple that it may be performed by inspection.

From these 27 second derivatives we may obtain by a simple formal differentiation any of the 64,800 second derivatives. Each of these second derivatives involves, besides certain of the ten original quantities

and the three fundamental first derivatives, only four independent second derivatives. Hence by eliminating these four second derivatives between the equations for any five second derivatives, we may obtain the relation between any combination of five second derivatives. There are approximately 9.5×10^{21} such relations.

The tables follow. These are given in three parts. First are the fundamental ten quantities with their equations of definition; second the tables in abbreviated form for the first derivatives; and lastly the 27 second derivatives necessary in obtaining any of the 64,800 second derivatives.

TABLE I.

The Fundamental Ten Quantities.

In this table are given the notation and the definition of the fundamental ten thermodynamic quantities. It is to be understood that all the quantities refer to unit amount of the substance. This unit is usually chosen either as 1 gm., or as the quantity that at 0° C. and atmospheric pressure occupies a volume of 1 c.c.

p = pressure per unit area.

τ = temperature on the absolute thermodynamic scale.

v = volume of the unit quantity of the substance.

s = entropy, defined by the integral, $\int dQ/\tau$.

Q = heat absorbed, measured in the mechanical units appropriate to p and v. A physical meaning can be given only to dQ, the heat absorbed during a given change.

W = work done by the substance, in the appropriate mechanical units. Here again, only dW has a physical meaning.

E = the internal energy of the substance in mechanical units. E may be changed by an additive constant without changing its physical meaning. E is one of the thermodynamic potential functions.

$H = E + pv$, the "total heat," also one of the potential functions.

$Z = E + pv - \tau s$, the Gibbs thermodynamic potential.

$\Psi = E - \tau s$, also a thermodynamic potential, the "free energy" of Helmholtz.

TABLE II.

The First Derivatives.

This contains the abbreviated notation by which any of the 720 first derivatives may be found in terms of $\left(\frac{\partial v}{\partial p}\right)_\tau$, $\left(\frac{\partial v}{\partial \tau}\right)_p$, and C_p. For instance, if $\left(\frac{\partial E}{\partial \tau}\right)_p$ is desired, write this in the equivalent form $\frac{(\partial E)_p}{(\partial \tau)_p}$,

and take the ratio of the functions given in the table for $(\partial E)_p$ and $(\partial \tau)_p$. Notice in general that $(\partial x)_y = -(\partial y)_x$.

(p)
$$(\partial \tau)_p = -(\partial p)_\tau = 1,$$
$$(\partial v)_p = -(\partial p)_v = (\partial v/\partial \tau)_p,$$
$$(\partial s)_p = -(\partial p)_s = C_p/\tau,$$
$$(\partial Q)_p = -(\partial p)_Q = C_p,$$
$$(\partial W)_p = -(\partial p)_W = p(\partial v/\partial \tau)_p,$$
$$(\partial E)_p = -(\partial p)_E = C_p - p(\partial v/\partial \tau)_p,$$
$$(\partial H)_p = -(\partial p)_H = C_p,$$
$$(\partial Z)_p = -(\partial p)_Z = -s,$$
$$(\partial \Psi)_p = -(\partial p)_\Psi = -[s + p(\partial v/\partial \tau)_p].$$

. . . . . .

(τ)
$$(\partial v)_\tau = -(\partial \tau)_v = -(\partial v/\partial p)_\tau,$$
$$(\partial s)_\tau = -(\partial \tau)_s = (\partial v/\partial \tau)_p,$$
$$(\partial Q)_\tau = -(\partial \tau)_Q = \tau(\partial v/\partial \tau)_p,$$
$$(\partial W)_\tau = -(\partial \tau)_W = -p(\partial v/\partial p)_p,$$
$$(\partial E)_\tau = -(\partial \tau)_E = \tau(\partial v/\partial \tau)_p + p(\partial v/\partial p)_\tau,$$
$$(\partial H)_\tau = -(\partial \tau)_H = -v + \tau(\partial v/\partial \tau)_p,$$
$$(\partial Z)_\tau = -(\partial \tau)_Z = -v,$$
$$(\partial \Psi)_\tau = -(\partial \tau)_\Psi = p(\partial v/\partial p)_\tau.$$

. . . . . .

(v)
$$(\partial s)_v = -(\partial v)_s = 1/\tau\{C_p(\partial v/\partial p)_\tau + \tau(\partial v/\partial \tau)^2_p\},$$
$$(\partial Q)_v = -(\partial v)_Q = C_p(\partial v/\partial p)_\tau + \tau(\partial v/\partial \tau)^2_p,$$
$$(\partial W)_v = -(\partial v)_W = 0$$
$$(\partial E)_v = -(\partial v)_E = C_p(\partial v/\partial p)_\tau + \tau(\partial v/\partial \tau)^2_p,$$
$$(\partial H)_v = -(\partial v)_H = C_p(\partial v/\partial p)_\tau + \tau(\partial v/\partial \tau)^2_p - v(\partial v/\partial \tau)_p,$$
$$(\partial Z)_v = -(\partial v)_Z = -\{v(\partial v/\partial \tau)_p + s(\partial v/\partial p)_\tau\},$$
$$(\partial \Psi)_v = -(\partial v)_\Psi = -s(\partial v/\partial p)_\tau.$$

. . . . . .

(s)
$$(\partial Q)_s = -(\partial s)_Q = 0,$$
$$(\partial W)_s = -(\partial s)_W = -(p/\tau)\{C_p(\partial v/\partial p)_\tau + \tau(\partial v/\partial \tau)^2_p\},$$
$$(\partial E)_s = -(\partial s)_E = (p/\tau)\{C_p(\partial v/\partial p)_\tau + \tau(\partial v/\partial \tau)^2_p\},$$
$$(\partial H)_s = -(\partial s)_H = -vC_p/\tau,$$
$$(\partial Z)_s = -(\partial s)_Z = -(1/\tau)\{vC_p - s\tau(\partial v/\partial \tau)_p\},$$
$$(\partial \Psi)_s = -(\partial s)_\Psi = (1/\tau)\{p[C_p(\partial v/\partial p)_\tau + \tau(\partial v/\partial \tau)^2_p] + s\tau(\partial v/\partial \tau)_p\}.$$

. . . . . .

(Q)
$$(\partial W)_Q = -(\partial Q)_W = -p\{C_p(\partial v/\partial p)_\tau + \tau(\partial v/\partial \tau)^2_p\},$$
$$(\partial E)_Q = -(\partial Q)_E = p\{C_p(\partial v/\partial p)_\tau + \tau(\partial v/\partial \tau)^2_p\},$$
$$(\partial H)_Q = -(\partial Q)_H = -vC_p,$$

$$(\partial Z)_Q = -(\partial Q)_Z = -\{s\tau(\partial v/\partial\tau)_p - vC_p\},$$
$$(\partial \Psi)_Q = -(\partial Q)_\Psi = p\{C_p(\partial v/\partial p)_\tau + \tau(\partial v/\partial\tau)^2{}_p\} + s\tau(\partial v/\partial\tau)_p.$$

. . . . . .

(W)
$$(\partial E)_W = -(\partial W)_E = p\{C_p(\partial v/\partial p)_\tau + \tau(\partial v/\partial\tau)^2{}_p\},$$
$$(\partial H)_W = -(\partial W)_H = p\{C_p(\partial v/\partial p)_\tau + \tau(\partial v/\partial\tau)^2{}_p - v(\partial v/\partial\tau)_p\},$$
$$(\partial Z)_W = -(\partial W)_Z = -p\{v(\partial v/\partial\tau)_p + s(\partial v/\partial p)_\tau\},$$
$$(\partial \Psi)_W = -(\partial W)_\Psi = -ps(\partial v/\partial p)_\tau.$$

. . . . . .

(E)
$$(\partial H)_E = -(\partial E)_H = -v\{C_p - p(\partial v/\partial\tau)_p\} - p\{C_p(\partial v/\partial p)_\tau + \tau(\partial v/\partial\tau)^2{}_p\},$$
$$(\partial Z)_E = -(\partial E)_Z = -v\{C_p - p(\partial v/\partial\tau)_p\} + s\{\tau(\partial v/\partial\tau)_p + p(\partial v/\partial p)_\tau\},$$
$$(\partial \Psi)_E = -(\partial E)_\Psi = p\{C_p(\partial v/\partial p)_\tau + \tau(\partial v/\partial\tau)^2{}_p\}.$$

. . . . . .

(H)
$$(\partial Z)_H = -(\partial H)_Z = -v(C_p + s) + \tau s(\partial v/\partial\tau)_p,$$
$$(\partial \Psi)_H = -(\partial H)_\Psi = -[s + p(\partial v/\partial\tau)_p][v - \tau(\partial v/\partial\tau)_p] + p(\partial v/\partial p)_\tau.$$

. . . . . .

(Z)
$$(\partial \Psi)_Z = -(\partial Z)_\Psi = -s[v + p(\partial v/\partial p)_\tau] - pv(\partial v/\partial\tau)_p.$$

. . . . . .

TABLE III.

The Second Derivatives.

In this table auxiliary formulas are given which simplify the computation of any second derivative in terms of the four following fundamental second derivatives; $\left(\frac{\partial^2 v}{\partial p^2}\right)_\tau$, $\frac{\partial^2 v}{\partial p \partial \tau}$, $\left(\frac{\partial^2 v}{\partial \tau^2}\right)_p$, and $\left(\frac{\partial C_p}{\partial \tau}\right)_p$. In general, if we desire to compute $\left[\frac{\partial}{\partial x_1}\left(\frac{\partial x_2}{\partial x_3}\right)_{x_4}\right]_{x_5}$, we write this in the form $\left[\frac{\partial}{\partial p}\left(\frac{\partial x_2}{\partial x_3}\right)_{x_4}\right]_{x_5} \times \left(\frac{\partial p}{\partial x_1}\right)_{x_5}$. But $\left(\frac{\partial p}{\partial x_1}\right)_{x_5}$ is already known, and $\left(\frac{\partial x_2}{\partial x_3}\right)_{x_4}$ is a known function of $\left(\frac{\partial v}{\partial \tau}\right)_p$, $\left(\frac{\partial v}{\partial p}\right)_\tau$, and C_p. Hence the result may be obtained by a purely formal differentiation, provided we know $\left[\frac{\partial}{\partial p}\left(\frac{\partial v}{\partial \tau}\right)_p\right]_{x_5}$, $\left[\frac{\partial}{\partial p}\left(\frac{\partial v}{\partial p}\right)_\tau\right]_{x_5}$, and $\left[\frac{\partial}{\partial p} C_p\right]_{x_5}$. These derivatives are listed in the following tables. If $x_5 \equiv p$, then we may use τ as our auxiliary variable of differentiation, and the entire differentiation may be performed by inspection.

. . . . . .

τ constant:

$$[\partial/\partial p\ (\partial v/\partial p)_\tau]_\tau = (\partial^2 v/\partial p^2)_\tau,$$
$$[\partial/\partial p\ (\partial v/\partial \tau)_p]_\tau = \partial^2 v/\partial p \partial \tau,$$
$$(\partial C_p/\partial p)_\tau = -\tau(\partial^2 v/\partial \tau^2)_p.$$

. . . . . .

v constant:

$$[\partial/\partial p\ (\partial v/\partial p)_\tau]_v = (\partial^2 v/\partial p^2)_\tau - \partial^2 v/\partial \tau \partial p \cdot (\partial v/\partial p)_\tau/(\partial v/\partial \tau)_p,$$
$$[\partial/\partial p\ (\partial v/\partial \tau)_p]_v = \partial^2 v/\partial p \partial \tau - (\partial^2 v/\partial \tau^2)_p \cdot (\partial v/\partial p)_\tau/(\partial v/\partial \tau)_p,$$
$$(\partial C_p/\partial p)_v = -\tau(\partial^2 v/\partial \tau^2)_p - (\partial C_p/\partial \tau)_p\ (\partial v/\partial p)_\tau/(\partial v/\partial \tau)_p$$

. . . . . .

s constant:

$$[\partial/\partial p\ (\partial v/\partial p)_\tau]_s = (\partial^2 v/\partial p^2)_\tau + \partial^2 v/\partial \tau \partial p \cdot \tau(\partial v/\partial \tau)_p/C_p,$$
$$[\partial/\partial p\ (\partial v/\partial \tau)_p]_s = \partial^2 v/\partial p \partial \tau + (\partial^2 v/\partial \tau^2)_p \cdot \tau(\partial v/\partial \tau)_p/C_p,$$
$$[\partial C_p/\partial p]_s = -\tau(\partial^2 v/\partial \tau^2)_p + (\partial C_p/\partial \tau)_p \cdot \tau(\partial v/\partial \tau)_p/C_p.$$

. . . . . .

Q constant:

The three derivatives at constant Q are identically equal to the corresponding three at constant s.

. . . . . .

W constant:

The three derivatives at constant W are identically equal to the corresponding three at constant v.

. . . . . .

E constant:

$$[\partial/\partial p\ (\partial v/\partial p)_\tau]_E = (\partial^2 v/\partial p^2)_\tau + \{\partial^2 v/\partial \tau \partial p\} \times \{\tau(\partial v/\partial \tau)_p + p(\partial v/\partial p)_\tau\}/\{C_p - p(\partial v/\partial \tau)_p,\}$$
$$[\partial/\partial p\ (\partial v/\partial \tau)_p]_E = \partial^2 v/\partial p \partial \tau + \{(\partial^2 v/\partial \tau^2)_p\} \times \{\tau(\partial v/\partial \tau)_p + p(\partial v/\partial p)_\tau\}/\{C_p - p(\partial v/\partial \tau)_p,\}$$
$$(\partial C_p/\partial p)_E = -\tau(\partial^2 v/\partial \tau^2)_p + \{(\partial C_p/\partial \tau)_p\} \times \{\tau(\partial v/\partial \tau)_p + p(\partial v/\partial p)_\tau\}/\{C_p - p(\partial v/\partial \tau)_p\}.$$

. . . . . .

H constant:

$$[\partial/\partial p\ (\partial v/\partial p)_\tau]_H = (\partial^2 v/\partial p^2)_\tau - \{\partial^2 v/\partial \tau \partial p\} \cdot \{v - \tau(\partial v/\partial \tau)_p\}/C_p$$
$$[\partial/\partial p\ (\partial v/\partial \tau)_p]_H = \partial^2 v/\partial p \partial \tau - \{(\partial^2 v/\partial \tau^2)_p\} \cdot \{v - \tau(\partial v/\partial \tau)_p\}/C_p,$$
$$(\partial C_p/\partial p)_H = -\tau(\partial^2 v/\partial \tau^2)_p - \{(\partial C_p/\partial \tau)_p\} \times \{v - \tau(\partial v/\partial \tau)_p\}/C_p.$$

. . . . . .

Z constant:

$$[\partial/\partial p\ (\partial v/\partial p)_\tau]_Z = (\partial^2 v/\partial p^2)_\tau + (v/s)\ \partial^2 v/\partial \tau \partial p,$$
$$[\partial/\partial p\ (\partial v/\partial \tau)_p]_Z = \partial^2 v/\partial p \partial \tau + (v/s)\ (\partial^2 v/\partial \tau^2)_p,$$
$$(\partial C_p/\partial p)_Z = -\tau(\partial^2 v/\partial \tau^2)_p + (v/s)\ (\partial C_p/\partial \tau)_p.$$

. . . . . .

$\Psi \quad [\partial/\partial p\,(\partial v/\partial p)_\tau]_\Psi = (\partial^2 v/\partial p^2)_\tau - \{\partial^2 v/\partial\tau\partial p\}\cdot\{p(\partial v/\partial p)_\tau\}/\{s + p(\partial v/\partial\tau)_p,\}$

constant:

$$[\partial/\partial p\,(\partial v/\partial\tau)_p]_\Psi = \partial^2 v/\partial p\partial\tau - \{(\partial^2 v/\partial\tau^2)_p\}\times \{p(\partial v/\partial p)_\tau\}/\{s + p(\partial v/\partial\tau)_p\},$$

$$(\partial C_p/\partial p)_\Psi = -\tau(\partial^2 v/\partial\tau^2)_p - \{(\partial C_p/\partial\tau)_p\}\times \{p(\partial v/\partial p)_\tau\}/\{s + p(\partial v/\partial\tau)_p\}.$$

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THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY,
CAMBRIDGE, MASS.

A STUDY OF THE PROPAGATION AND INTERCEPTION OF ENERGY IN WIRELESS TELEGRAPHY.[1]

BY CHARLES A. CULVER.

PART II.

KIEBITZ and others have shown that it is possible to utilize horizontal antenna placed close to the earth's surface in practical radio-telegraphy.

The purpose of this investigation is to compare the absorbing and radiating efficiency of a simple vertical antenna with certain horizontal types.

As a standard of comparison, an antenna was erected on the college campus consisting of a single vertical wire 21 m. in length. Number 14 aluminum wire was used for this and the other antennæ in these experiments. A standard earth connection was also established. This consisted of four holes located on the cardinal points and one meter deep. The holes were placed two meters apart and were 25 cm. in diameter. A cylindrical piece of galvanized iron netting enclosing charcoal was placed in each hole. This filled the holes to within 40 cm. of the top. Earth completed the filling. All four earths were connected together by a heavy copper wire. The soil in this vicinity consists largely of sand and gravel.

The horizontal aerials referred to later were supported by small porcelain insulators attached to wooden stakes, these stakes being just high enough to prevent the wire from touching the grass.

A thoroughly equipped private station belonging to Mr. Hiram Morgan and located approximately 5 km. from the Beloit campus was utilized as a coöperating station. The Morgan transmitting equipment consists of a *T* aerial, the horizontal portion consisting of six wires 21 m. in length spaced 60 cm. apart. A single wire 12 m. in length constituted the vertical part. The horizontal portion is approximately parallel to the direction of *B*. Energy from a 1-kw., 60-cycle transformer and non-synchronous rotary gap was supplied to the above aerial through a loose coupling. The wave emitted by this station is 220 m., with an antenna current of 250 milliamperes.

[1] For Part I., see PHYS. REV., Vol. XXV., p. 200, Series I., September, 1907.

The plan of the tests consisted in having the Morgan station send long dashes for a period of a half hour daily from 9:00 to 9:30 A.M. The various horizontal aerials were then compared with the standard and with one another by means of the shunted telephone method. While more or less large variations occurred in the data, it was possible to obtain concordant results by repeating given tests on different days. These experiments were carried out between August 1 and September 15. Fig. 1 shows the form and dimensions of the various antennæ which were compared. The first series of tests consisted of a comparison

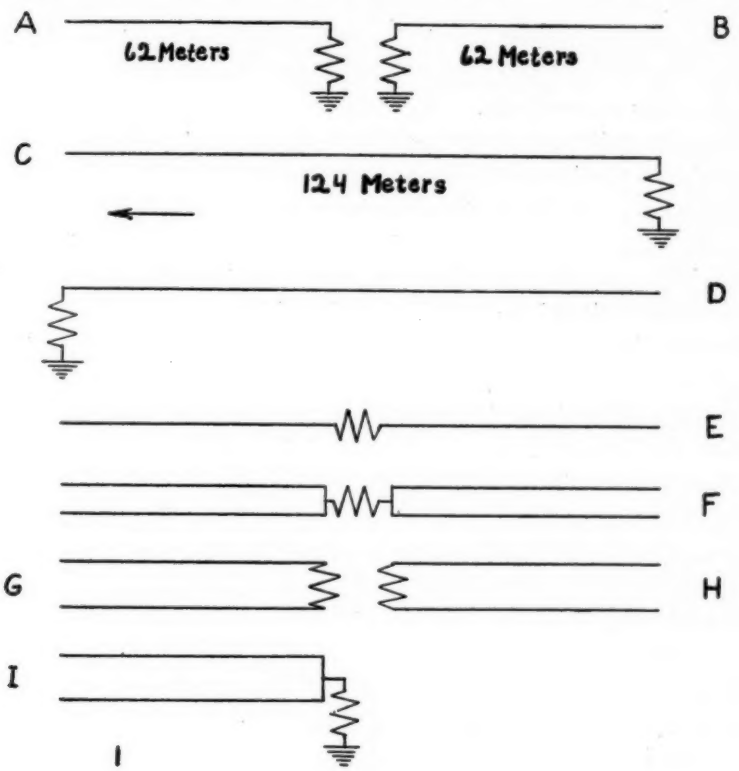


Fig. 1.

In the above diagrams the arrow indicates the direction of the Morgan station. The inductance shown is the primary of the receiving transformer.

between systems *D* and *C*. The standard earth connection was utilized in this experiment. Type *C* gave the higher efficiency, the ratio of the resistances shunted about the receivers being 8/3. System *E* gave approximately the same efficiency as *C*. Using system *D* and the standard earth, the effect of grounding the free end of the aerial directly and through a condenser (.0015 m.f.) was tried. The signals were barely audible with both of these arrangements.

On another date a comparison was made between system *C* and the standard vertical aerial. The efficiency of these two systems proved to be practically the same. System *D* showed a lower efficiency than the standard, the ratio of the shunt resistances being approximately 1/5. During the tests just mentioned the ground was very wet.

A test was next made of the effect of grounding the free end of system *A*, both directly and through a condenser (.0015 m.f.). Little if any difference in the efficiency was found to exist whether the system was free or grounded by either method. This test was made while a light rain was falling.

A comparison was next made between systems *C* and *E*. System *C* proved to be more efficient than *E*, the shunt resistances being 185 ohms and 340 ohms respectively.

Comparisons between systems *F* and *E* appeared to indicate that the single wire of the latter system was practically as efficient as the double wire system *F*. At times *F* gave a slightly higher efficiency.

Upon testing systems *G* and *H* it was found that the signals were just audible, but too weak to permit measurements being taken. Comparisons between *E* and *A* and *B* showed that *A* and *E* gave practically the same efficiency, while *B* fell somewhat below *E*, the ratio of the shunt resistances being 5/3.

Repeated comparisons between *E* and the standard showed that while at times the standard gave a slightly higher efficiency than *E*, yet on other days the efficiency was practically the same.

System *G* was so arranged that one of the two wires lay directly on the ground while the other was insulated in the usual manner. No signals could be detected with this arrangement. However, when system *E* was so arranged that the wire lay directly on the grass, readable signals were secured.

The next tests consisted in an effort to utilize a capacity area as a receiving system. The capacity area consisted of a galvanized iron screen one by two meters and supported by suitable insulation two meters above the earth. No signals could be heard with this arrangement. The screen was then placed in a vertical position with the upper edge two meters from the earth, the plane of the screen being at right angles to the line joining the stations. The result was nil in this case also.

In the following described tests the Beloit station operated as a transmitting equipment and the Morgan station received the signals. When receiving the Beloit signals, the Morgan station utilized a T aerial consisting of six horizontal wires 91 m. in length spaced 60 cm. apart, and

vertical leads 12 m. in length. The horizontal section is parallel to the direction of Beloit. This aerial was connected to the usual loosely coupled receiving equipment in series with the primary of which was inserted a Duddell thermo-galvanometer for the purpose of measuring the intercepted energy. The Beloit transmitting equipment consisted of a 1 kw. transformer operated through a fixed gap, a typical set of connections being indicated in Fig. 2. A series of tests was made in an effort to determine the relative efficiencies of the above mentioned horizontal aerials as radiating systems. A wave length of 300 meters was used in these latter tests, except with types *C* and *D*, this being the shortest wave which could be advantageously employed with this particular transformer. In order to strongly excite systems *C* and *D* it was necessary to employ at least a 780 m. wave. The results of the radiation experiments are shown in the following table.

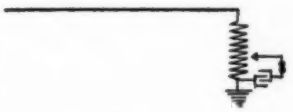


Fig. 2.

Test Number.	Type of Aerial.	Conditions.	Current in Aerial (Approximate.)	Current at *M.* Scale Units.
I	Std.	Std. earth		2.0
	A	Std. earth	0.65 amp.	2.2
	B	Std. earth	0.6	1.2
II	Std.	Std. earth	0.25	0.8
	C	Std. earth	0.6	7.5
	E	No earth	0.4	4.5
III	*I*	Std. earth	0.3	1.0
	G	No earth	0.5	0 0
	A	Std. earth	0.3	1.2
	E	No earth	0.3	3.5
	D	Std. earth	0.35	2.5
IV	*A*	Std. earth and free end grounded through condenser	0.25	1.0

Recently we have received signals at this laboratory when utilizing a pair of number 12 galvanized iron wires in parallel extending between the college observatory and Pearson's Science Hall. This local telephone line is approximately 250 m. in length and about 4 m. above the ground. The line, while somewhat curved, has in general a southerly direction. Several stations at different points along the Atlantic coast have been identified when using this line as a receiving aerial, among which might be mentioned Key West and Cape Cod. Further long distance tests of

this character are in progress at the present writing, the results of which will be reported shortly.

DISCUSSION.

The data secured from experiments carried out in a particular geographical locality do not warrant general conclusions. However, certain of the above results are at least significant. The experiments of the author confirm and extend the tests made by Kiebitz and others, and would appear to indicate that horizontal antennæ placed very near the earth's surface exhibit efficiencies both as receiving and radiating systems which are comparable with a simple vertical aerial.

The data would also appear to indicate that when utilizing such low horizontal antennæ either as absorbing or radiating systems, the free end of the aerial should point in the direction of the second station. This appears not to conform to Marconi's law of horizontal antennæ and may be due to local topographical conditions.

The fact that energy was intercepted and radiated when both ends of the horizontal systems were grounded would appear to indicate that we are dealing in such cases with what Sommerfeld calls a surface wave, as more or less distinct from a space wave. This would also appear to be borne out by the results obtained with system *G*, and by the fact mentioned above, namely, that the receiving and exciting equipment operated most effectively when located at the remote end of the aerial. The amplitude of the surface wave, which acts to develop a difference of potential between the ends of the horizontal wire, would be greatest at the remote end when that end pointed toward the second station. These conditions would result in a maximum effect on a potential operated device such as the crystal detector used in the first part of the above experiment when connected as in systems *A* and *C*. An examination of the above data will show that the empirical results are in conformity with this explanation.

In conclusion the writer has to thank Professor Goodspeed, of the University of Pennsylvania, for the loan of valuable apparatus used in certain of the above experiments. We are also deeply indebted to Mr. Hiram Morgan, of this city, for his careful and faithful coöperation in carrying out these experiments.

BELOIT COLLEGE,
January 1, 1914.

ON THE CONTINUED APPEARANCE OF GASES IN VACUUM TUBES.

By George Winchester.

IN undertaking any investigation where a high vacuum is necessary the phenomenon of "occluded gases" is encountered, that is, when a high potential is applied to the electrodes, hydrogen, helium and neon are freed and apparently keep coming out of the electrodes for a considerable length of time. Some discussion has arisen concerning the origin of these gases. One idea suggested is that the gases are simply occluded and that the impact of the cathode rays against the anode frees them and they escape into the tube; another is that they are occluded, but instead of being freed at the anode by the bombardment of cathode rays, they are freed at the cathode by reason of the high potential placed upon the electrodes; still another notion is that the material of which the anode or cathode is made disintegrates in somewhat the same manner as uranium does and that these gases are the product of the disintegration process. E. Cohnsteadt,[1] working with a differential method, aims to eliminate the gases which may emanate from the glass walls of the tube and to measure only the gases evolved from the electrodes. A close relation between the amounts of gases given off and the amount of water vapor present is established. He then concludes that the gases are not evolved from the metal electrodes but are due entirely to water vapor on their surfaces and that the amounts of gases produced do not depend upon the volume of the electrodes but upon their area. He uses P_2O_5 to free his tubes from moisture.

Sir William Ramsay, Professor Collie and Mr. Patterson[2] described some experiments which they believe to prove the transmutation of other elements into helium and neon. These experiments consist simply in applying high potentials to the electrodes in vacuum tubes; they further suggest that helium (4) and oxygen (16) may combine to form neon.

Sir J. J. Thomson, using the positive ray method, has shown the presence in vacuum tubes of hydrogen, helium and neon, and another gas, which he calls X_3. He concludes that these gases are given out by

[1] The Water-Film on Glass and Aluminium, and its Influence on the Pressure in Vacuum Tubes, Ann. d. Physik, 38, 1, pp. 223–238, May 7, 1912.

[2] Report in Nature, Vol. 90, p. 653, February 13, 1913.

the different materials which were subjected to the bombardment of cathode rays.[1] Professor Skinner[2] in a series of experiments on the evolution of hydrogen in gases shows that gases may be absorbed by the metal in a vacuum tube as well as given out; he also describes experiments to show that the cathode itself is the sole source of the supply of hydrogen.

My attention was first directed to this evolution of gas about ten years ago while preparing tubes for work in ultra-violet light. Several years ago the attempt was made to find out whether or not the electrodes could be entirely denuded of gases. At that time, not knowing the large quantity of gases contained in metals nor the length of time consumed in getting it out, I used electrodes of aluminium 2 mm. in diameter and 7 cm. in length with the free ends formed into rings facing each other coaxially at 1 mm. distance. I found this method of placing the electrodes most fertile in the production of gas, for at extremely low pressures I could place very large potentials on the electrodes when they were close together and force the gas out at a more rapid rate. Using this method the experiment must be watched or the electrodes will bridge.

One thing noticeable in tubes of this kind is that, whereas some yield only comparatively small amounts of helium, others are very rich in this gas. One tube which at first gave only a small amount of helium in comparison with hydrogen, suddenly after running for fifteen days gave out an enormous amount of helium for a few days and then just as suddenly became normal again. The pump was worked on this tube so that the pressure never rose much above .3 mm. Since then I have found two tubes that showed the same phenomenon. Some electrodes seem to be very rich in helium and some very poor. All of the aluminium electrodes mentioned in this paper were made of c.p. aluminium unless otherwise stated.

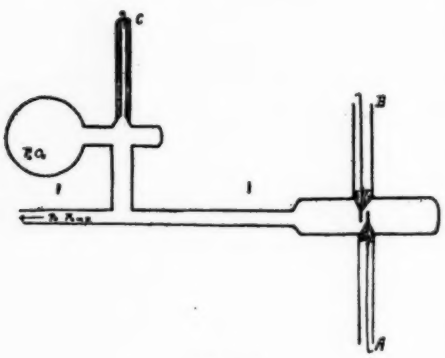


Fig. 1.

During the last year I have again attempted to denude electrodes of their gases by using very small electrodes, placing them close together, and applying high potentials. In these experiments as well as in the

[1] Some Further Applications of the Method of Positive Rays, Nature, Vol. 91, p. 333, May 29, 1913.

[2] The Evolution of Hydrogen from the Cathode and its Absorption by the Anode in gases, PHYS. REV., Vol. XXI., p. 1, July, 1905.

first ones, the pumps used were cleaned thoroughly with a hot saturated solution of potassium bichromate in concentrated sulphuric acid, rinsed with distilled water and then with c.p. alcohol; the tubes were cleaned in the same manner and sealed directly to the pump; a small bulb containing P_2O_5 was also directly sealed to the pump. Fig. 1 shows the arrangement in diagram. Usually before any account was taken of the gases given off the tube was heated for three or four hours with a bunsen flame and the coil run from ten to twenty hours. The tube was heated to the highest temperature before the pump was started so that if softening of the glass took place the tube would not collapse.

The presence of stop-cock grease has apparently nothing to do with either the amounts or kinds of gases that are given off. For example, in one experiment especial care was taken in cleaning the pump and tube to eliminate hydrocarbons; the bulb was heated almost to melting to drive off water vapor, leaving as nearly as possible nothing but glass and the c.p. aluminium electrodes. This tube was run for several months during which time the gases were measured and examined with the spectroscope and photographic plate. The tube was then cut off, a stop-cock sealed on to the pump and to this a tube containing commercial aluminium electrodes of the same size as the chemically pure electrodes. No attempt was made to clean this tube more than to heat it while the process of exhaustion was going on. The gases were measured and examined by the photographic plate; exactly the same gases were given off from the two sets of electrodes in approximately equal amounts.

Analysis, by means of the spectroscope, of the gases produced shows the presence of hydrogen, helium and neon. If the gases which are given off first are pumped out into an examination tube the helium and neon spectra are quite prominent. If the gases given off are examined in the same manner, after running the tube for a month, these two spectra have entirely disappeared. Seven different tubes were thus denuded of all traces of either helium and neon, as determined by photographic methods. This supports the theory that the helium and neon are simply occluded and not transmuted from some other element. For if these gases are produced by the disintegration of some element because of the application of a large voltage, we should be able to produce them so long as conditions are kept constant within the tube; that is, as long as there is any electrode, the continuous application of a high potential should produce helium and neon. Thomson[1] describes an experiment in which he sparked iron electrodes in an atmosphere of 3 cm. of hydrogen for an hour or so each day for four days; at the end of that time not a trace of helium or

[1] Loc. cit., p. 6.

neon could be found, even by the positive ray method. In my experiments I have never been able to eliminate these gases so quickly from aluminium electrodes; using one aluminium electrode of 10 mg. weight as cathode with a platinum electrode of about 5 mg. weight as anode, helium has been discovered after several weeks of sparking almost constantly, *i. e.*, say, in the neighborhood of eight hours a day. I have noted that when the pressure within the apparatus becomes at all appreciable the gases are very slow in coming out; and it may be that in the presence of 3 cm. of hydrogen the gases refuse to come out except from the surface layer. The case of hydrogen is different; I have sparked tubes until the electrodes were entirely wasted away and this gas can be obtained as long as any metal remains.

Skinner's[1] experiments show that after the current has flowed for a short time "the pressure appears to have reached a constant maximum value." This seems to be true unless the coil is run for a considerable length of time, since the ordinary McLeod gage does not show very small differences in pressure. Fig. 2 shows this phenomenon when the poten-

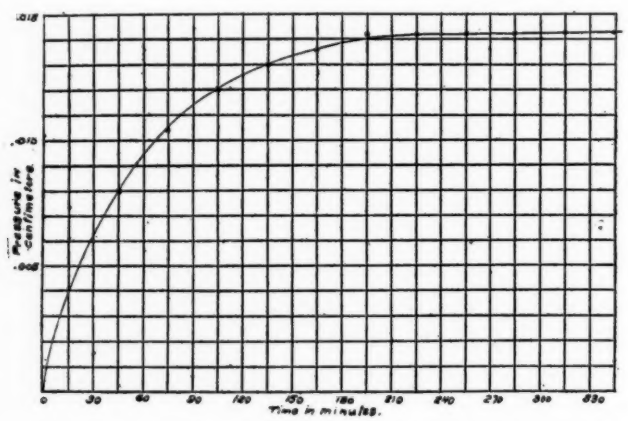


Fig. 2.

tial applied is approximately 100,000 volts. Here the current is continued for two hours after the apparent maximum has been reached and the curve shows a slight rise in pressure. The gage used in determining these pressures magnified them about 150 times. That there is not a maximum pressure, but merely an elbow, and that the gas is evolved at a very slow but constant rate, is shown when the gage used is more sensitive. Figure 3 shows the increase in pressure with the time when the pressures within the tube have been magnified about three thousand times. Of

[1] Loc cit., p. 6.

course the position of this elbow depends upon the value of the potential applied to the electrodes as will be shown farther on. Undoubtedly this apparent discrepancy in the value of the pressure is explained by the unreliability of Boyle's law for very small pressures as determined by the McLeod gage. Since the curves shown by Professor Skinner represent experiments upon the evolution of gas in helium at 3 mm. pressure,

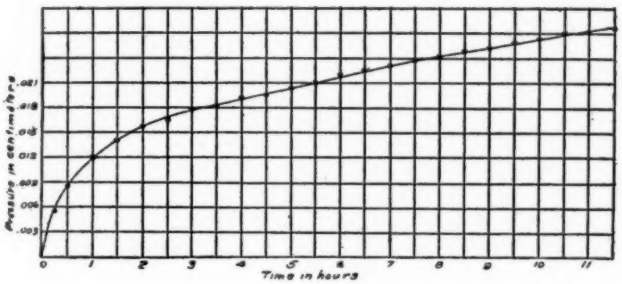


Fig. 3.

while Fig. 2 represents the increase in pressure starting at a pressure of perhaps one-millionth of a millimeter, the position of the elbow seems not to be determined by the actual pressure within the tube but perhaps by some function of the relation between the amount of gas evolved and the amount still remaining within the surface layer.

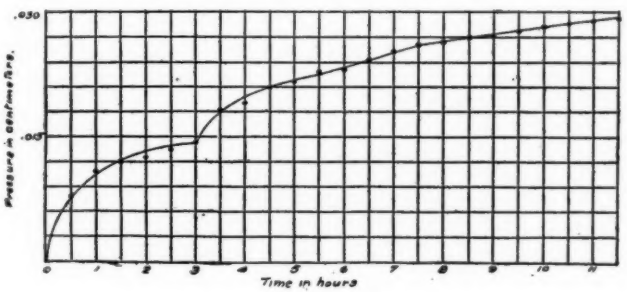


Fig. 4.

It may be observed under different conditions; for example, Figure 4 shows the increase in pressure taking place in the usual manner for three hours; then the tube was rested for eighteen hours and the coil started again. Here again the gas flows very readily at the start and the curve rises very rapidly for a time and then settles down to a constant slope. Fig. 5 is an analogous curve showing what happens when the potential on the electrodes is suddenly increased. The gas is evolved copiously for a short time when the increase in potential is made. Fig. 6 represents

the increase in gas pressure which takes place under the following conditions: on the first part of the curve the coil is run steadily for fifteen hours showing a very steady flow of gas; at the end of this time all the gas is quickly pumped out (requiring forty-six minutes, in this instance) and the coil started again. This is apparently equivalent to resting the electrode or raising the potential. The gas is evolved in the same manner as at the beginning of the experiment, only not so profusely.

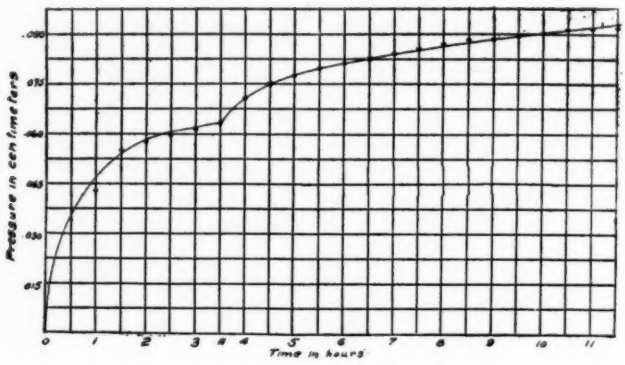


Fig. 5.

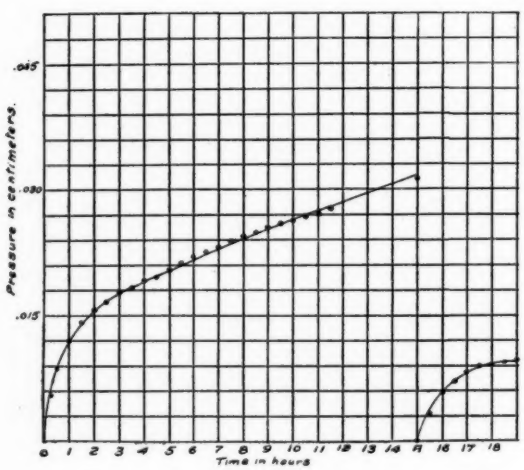


Fig. 6.

CONCLUSIONS.

1. Chemically pure electrodes give off gases in different proportions; for example, of two cathodes of the same weight one may be rich in helium and the other give off comparatively little; or one may give off

more hydrogen than the other. Thus, there is apparently no relation existing between the size of the cathode and the amounts of gases which may be thrown off in a high vacuum when large potentials are applied to the electrodes by means of an induction coil. The explanation of this fact may depend upon the manner in which the gases are occluded in the metal. It seems probable that they are not a part of the metallic molecule in the same sense that the α particle is a part of the molecule of radium. Suppose, for instance, that the gaseous atom is simply held in contact with the aluminium atom or molecule by some force of adhesion, and that it is freed by repulsion on account of the very large number of negative electrons applied to the cathode. Then if one cathode should disintegrate more thoroughly than the other before it was consumed, it would give off the more gas.

2. The presence or absence of hydrocarbons within the experiment tube has no influence upon the amounts or kinds of gases evolved. Thus, after having used all precautions in removing hydrocarbons from the tube the experimental results are similar to those obtained when a stopcock is used close up to the experiment tube. If, as some believe, the hydrogen comes from the moisture within the tube, there should be some indication of the liberated oxygen, for in such a dry atmosphere there should be absolutely no reaction between either oxygen and hydrogen and the phosphorous pentoxide.

3. So far as these experiments have gone there has been no instance when hydrogen could not be obtained if any metal of the cathode remained; with this exception: a cathode of 10 mg. of aluminium when about three-fourths disintegrated suddenly stopped giving out gas and remained thus for fourteen hours. That is, the pressure did not increase during the last fourteen hours the coil was running. At that time it was discovered that the coil was acting very poorly and upon testing gave a comparatively low potential. Before the coil could be repaired and the experiment continued the tube cracked. It seems very improbable, though, that this tube should behave differently from the others. Apparently, then, hydrogen is evolved as long as any metal remains in the tube.

4. The fact that helium and neon may be entirely eliminated from the tube shows that they are not disintegration products; if they were we should be able to generate them as long as we apply high potentials to the electrodes. The fact that these two gases may be eliminated from the metal before a very appreciable part of it has disintegrated might indicate that they were occluded at or near the surface. If this is true, it may be that the helium and the neon are absorbed from the atmosphere.

This would account for the presence of more helium or neon in one electrode than in another because the previous history of the metal would somewhat determine how much of a certain gas had been absorbed.

But the case of hydrogen is different. There is the possibility of its being a disintegration product of the metal in somewhat the same manner as the α particle is disintegrated from radium. But while radium disintegrates spontaneously, the hydrogen requires the assistance of a large electric force before it is able to leave the metal.

PHYSICAL LABORATORY OF WASHINGTON AND JEFFERSON COLLEGE,
February 20, 1914.

THE MAGNETIC SUSCEPTIBILITY OF WATER.

BY H. C. HAYES.

THE magnetic susceptibility of water has been a subject of investigation by many physicists among whom, excepting Townsend who used a method of induction, three methods have been employed, each based on the fact that a substance is acted on by a force when it is placed in a non-uniform magnetic field. I shall speak of these methods as the method of Faraday, the method of Quincke, and the method of Wills.

The Method of Faraday.—This method is based on the well-known law,

$$F_s = XMH\frac{dH}{dS},$$

where F_s is the force which the specimen experiences in the direction S, X is the specific susceptibility, M is the mass of the specimen, H is the value of the field in the space occupied by the specimen, and dH/dS is the space derivative of the field in the direction S. The method is useful for comparing the susceptibility of different substances, but has been discarded as a means of getting absolute values, because of the difficulty in measuring the factor dH/dS and also because of the error that may arise from the assumption that this derivative is constant through the space occupied by the specimen.

The Method of Quincke.—If a liquid has two free surfaces one of which is in a magnetic field of strength H and the other in a null field, a difference in level, d, will exist between the two surfaces such that

$$d = X\frac{H^2}{2g},$$

where g is the acceleration due to gravity. This method requires the measurement of but two factors and lends itself to exact experimentation.

The Method of Wills.—If a uniform cylinder of a substance has one end placed in a magnetic field of strength H_1 and the other end in a weaker part of the field H_2, the cylinder will experience a force, F, in the direction H_2H_1 such that

$$F = {}^1/_2XDS[H_1^2 - H_2^2],$$

D and S being the density and cross-section respectively of the cylinder. This method also admits of exact experimentation, but the factor, F,

like the factor d in Quincke's method, is very small when a substance as weakly magnetic as water is used. As a result some experimenters have determined the value of X for a para-magnetic substance by one of the last two methods and then found the value for water in terms of this by Faraday's method.

The following table gives the results obtained by various investigators and the method employed.

Date.	Experimenter.	Method.	Temperature.	$-X \times 10.^7$
1855	Faraday	Faraday	—	6.7
1880	Becquerel	Faraday	—	6.4
1881	Schumeister	Faraday	—	4.19–5.25
1885	Quincke	Quincke	20	8.1
1888	Wahner	Quincke	15	5.3
1888	Du bois	Various	—	8.3
1892	Henrichsen	Wills, Faraday	—	7.24
1895	Curie	Faraday	15–159	7.87
1896	Townsend	Induction	10	7.4
1898	Fleming, Dewar	Faraday	20	7.4
1898	Konigsberger	Quincke, Faraday	20	6.7
1899	Jager, Meyer	Quincke Wills, Faraday	13–30	6.4
1902	Piaggesi	Quincke	23–86	7.8
1903	Stearns	Wills	22	7.06
1904	Wills	Wills	22	6.95
1912	Sève	Quincke	24	7.2
1912	de Haas, Draper	Quincke	24	7.3
1912	Weiss, Piccard	Quincke, Faraday	20	7.193
1913	Hayes	Faraday	24	7.26

The present investigation was begun in 1911 for the purpose of obtaining an accurate standard to be used in an investigation of the magnetic properties of some weakly magnetic alloys. The work was well under way when the papers by Sève, de Haas and Draper, and Weiss and Piccard came out in succession. As the table shows, their results are in good agreement, so this paper which offers results in excellent agreement with theirs is valuable only as a piece of corroborative evidence.

Method.—The present work has made use of Faraday's method. As shown above, this method is based on the law

$$F_s = XMH\frac{dH}{dS}$$

and the experimental difficulties, so far as making absolute determinations is concerned, lie in making accurate measurements of the factor dH/dS. Much of the error in the work that has been done by this method has

been due to the fact that the specimen was not located at the position where the value of $H(dH/dS)$ was determined. The author has devised an accurate method for determining the factor, dH/dS, and the design of the apparatus eliminates any error due to misplacing the specimen, so the method has been made suitable for determining absolute values.

Apparatus.—The field H was furnished by an electro-magnet MM (Fig. 1), which was suspended from a turn-table provided with ball bearings.

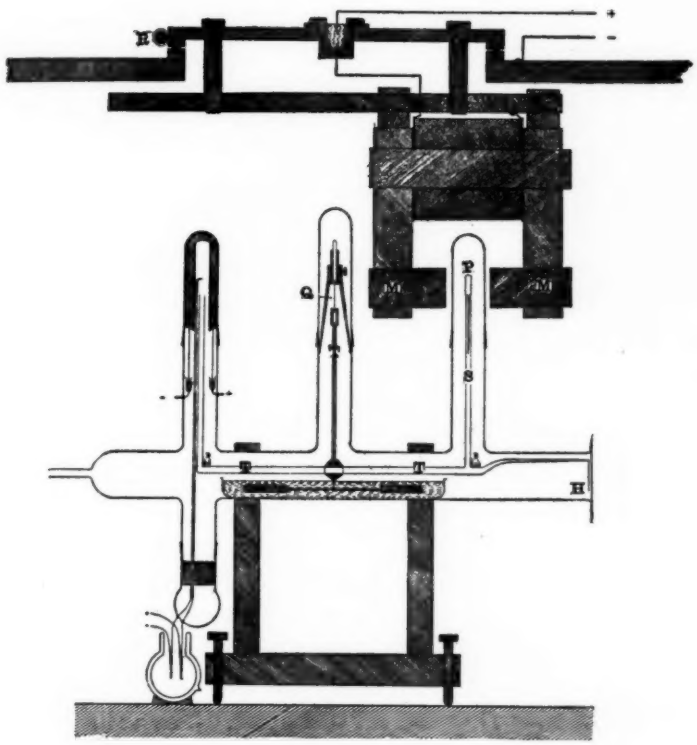


Fig. 1.

Arrangement of apparatus. MM, electro-magnet; R, worm gear, TTT, torsion balance; S, standard for supporting specimen; P, specimen; B, mercury-cup for counterbalance; Q, quartz fiber, H, cross-hair.

A worm gear R, attached to the rim of the turn-table, made it possible to rotate the magnet to any desired position about a vertical axis, one complete turn of the magnet requiring 793 turns of the worm. To avoid waste of time in moving the magnet, an arrangement was made whereby the worm could be turned either by a motor or by hand. This arrangement is not shown in the figure.

The magnet was made of wrought iron and carefully annealed. It

was excited by a current of five amperes taken from a storage battery of twenty cells. The current was controlled by means of a slide-wire resistance.

The specimen P was placed on the standard S erected on one arm of a torsion balance TTT, and counter-balanced by means of the mercury cups BB. The torsion fiber Q, the torsion arm, and the standards were made of quartz. The balance was damped by a pair of vanes submerged in mercury.

The standard from the left arm of the balance enters a small electric furnace provided with a thermo-junction for measuring the temperature. This part of the apparatus plays no part in the present investigation but is used for making alloys and testing their magnetic properties. When the apparatus is used for this purpose the right-hand arm carries the standardizing specimen.

The whole balance is suspended in glass, and all measurements were made in a vacuum varying from 0.002 to 0.003 of a centimeter of mercury. This method has some advantages over the usual one of surrounding the specimen with a so-called magnetically indifferent gas.

Measurements.—The force F_s was determined through the relation

$$F_s = \frac{T}{L \times l} \times d,$$

where L is the length of the cross-hair arm, l is the length of the torsion arm, T is the coefficient of torsion of the quartz fiber, and d is the deflection of the cross-hair H. The values of L and l, as determined by a cathetometer, were 29.66 and 21.64 centimeters respectively. T was determined experimentally by observing the time in which a bar of known moment of inertia made one complete oscillation when it was suspended by the fiber Q at the central point. The value of T proved to be 1,208 so the relation between F_s and d was

$$F_s = 1.882d.$$

The deflection, d, measured with a microscope and micrometer eye-piece, was taken as the maximum value given by moving the magnetic field slowly across the specimen. It follows that the corresponding values of H and dH/dS must be such as to make their product a maximum. This method eliminates any error that might arise from misplacing the specimen, and avoids disturbing the specimen by turning the torsion head.

The Maximum Value of dH/dS.

First Method.—The value of H was determined at various points across the field corresponding to equal increments of S, and from this data the curve H^2 vs. S was carefully plotted. The slope of this curve is equal to twice the product of H and dH/dS, so the maximum value of this product is half the maximum value of the slope of this curve. As shown is Fig. 2, this particular value of the slope can be determined with considerable accuracy, for it occurs at a point of flection.

The increments in S were taken as 0.172 cm., the distance the field moves across the exploring-coil when the worm is given one complete turn. H was determined by means of a ballistic galvanometer through the relation,

$$\frac{AH}{MI} = \frac{\theta}{\theta_s},$$

where A is the area of cross-section of the exploring-coil, H is the field strength in gausses, M is the mutual induction of the standard in gausses per ampere of current, θ is the deflection of the galvanometer caused by suddenly turning the exploring-coil through 180°, and θ_s is the deflection caused by reversing a current of I amperes in the primary of the standard.

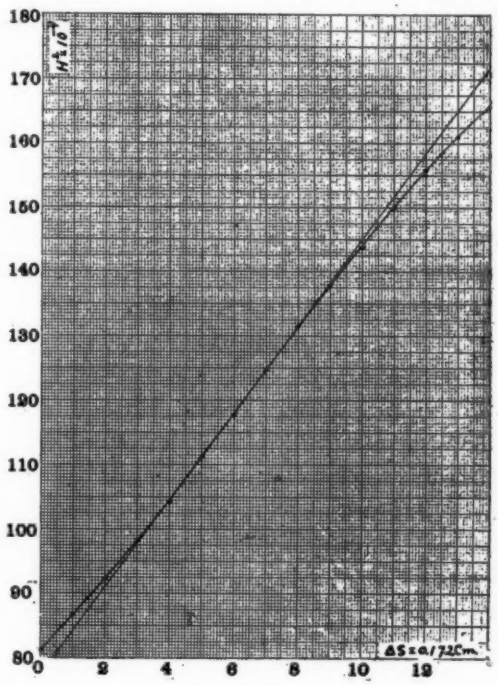


Fig. 2.

Portion of H^2 vs. S curve showing tangent drawn at point of deflection.

By varying the standardizing current I, the value of θ_s was always made nearly equal to θ; and the deflection θ was held to about 10 cm. by varying the resistance of the galvanometer circuit.

The exploring-coil was 2.5 cm. long and 0.35 cm. in diameter, being cylindrical in form and of the same cross-section as the specimen. This coil was made of No. 40 double silk-covered wire wound on a thin strip of mica and covered with bees-wax. It was mounted vertically on a standard that could be rotated through 180° about a vertical axis, and

adjusted so that it passed across the same portion of the field, when the magnet was rotated, as did the specimen.

The area of the coil was determined by balancing it against a coil of known cross-section and so varying the turns on this coil that a sensitive galvanometer connected in series with the two coils gave no deflection when the field was reversed.

Using the method described, the maximum value of $H \cdot dH/dS$ was determined four separate times with the following results:

2.00×10^5
2.04 " "
2.02 " "
2.03 " "

Average, 2.02×10^5

Second Method.—Here both factors H and dH/dS were measured separately. H was determined as in the first method, but the method for evaluating dH/dS is, I think, used for the first time. It consisted in rotating the magnetic field across the exploring coil at a uniform speed and measuring the E.M.F. developed by the change of flux through this coil.

The magnet was rotated uniformly by turning the worm with a motor which had a heavy fly-wheel attached to its axis. The inertia alone was sufficient to keep the motion uniform to within 1 per cent. while the magnet traveled the required distance.

The exploring-coil was connected in series with an Einthoven galvanometer which recorded on a photographic film the E.M.F. drop across the terminals of this instrument. Then the total E.M.F. developed in the coil was found by the relation

$$\frac{E}{e} = \frac{R}{r},$$

where E is the E.M.F. developed in the coil, e the E.M.F. recorded by the galvanometer, R the resistance of the circuit, and r the resistance of the galvanometer.

The recording film and the magnet were operated by the same motor, so it was only necessary to know what point in the magnetic field corresponded to the beginning of the E.M.F. curve to make the relation between field and curve complete. The shutter for exposing the film, and the pen on a chronograph were operated by electro-magnets connected in series in a circuit containing two storage cells. When this circuit was closed the film was exposed and the pen held down, thus giving the time of exposure. A mercury-trough, attached to the outside

arm of the magnet and embracing the portion of S which was to be investigated, closed this circuit by short circuiting two amalgamated points.

This arrangement always started the exposure at the same position of the magnet, but variation in the spark when the mercury passed from beneath the points, caused a slight change in the length of exposure. However, the distance the magnet traveled during the time of exposure was always given by dividing the length of exposure on the film by the ratio between the motion of the film and magnet. The velocity of the field across the exploring coil was given by dividing the distance the field moved across the coil during an exposure by the time of exposure.

From the record of the E.M.F. developed in the exploring-coil as the field moves across this coil with a uniform velocity, V, the value of dH/dS can be determined for any position along the path traversed by the coil as follows:

$$dS = V \cdot dt$$

where dt is the increment of time in which the field moves a distance dS. Also,

$$E = d\Phi/dt = A \cdot dH/dt,$$

where Φ is the flux through the coil and A is the area of the coil. Substituting for dt its value, dS/V, we get the relation,

$$dH/dS = E/A \cdot V.$$

The value of dH/dS for any position along the path S is given by taking the value of e at the corresponding point on the E.M.F. curve, dividing this value by $A \cdot V$, and multiplying this by R/r.

The procedure for determining the maximum value of $H \cdot dH/dS$ is shown in Fig. 3. Curve 1 is a reproduction of the photographic record taken by the Einthoven galvanometer when the magnetic field was rotated across the exploring-coil. The ordinates of this curve gives the deflection of the Einthoven and the abscissæ represent distance along the path S.

From this curve and the calibration curve of the galvanometer we get Curve 2, having for ordinates the E.M.F. drop across the terminals of the galvanometer. From Curve 2, through the relation,

$$dH/dS = e \cdot R/A \cdot V \cdot r,$$

Curve 3 is obtained having for ordinates dH/dS. Curve 4 gives the value of H through the region where dH/dS has been determined. As stated, this curve is obtained from data given by the ballistic galvanometer. Curve 5 has for its ordinates the product of the corresponding ordinates of Curves 3 and 4. The maximum of this curve gives the maximum

value of $H \cdot dH\,dS$. It should be stated that the ordinates in Curve 3 are only proportional to dH/dS for the factor R/r was omitted in order to simplify the computation. Therefore, the maximum ordinate in Curve 5 must be multiplied by this factor.

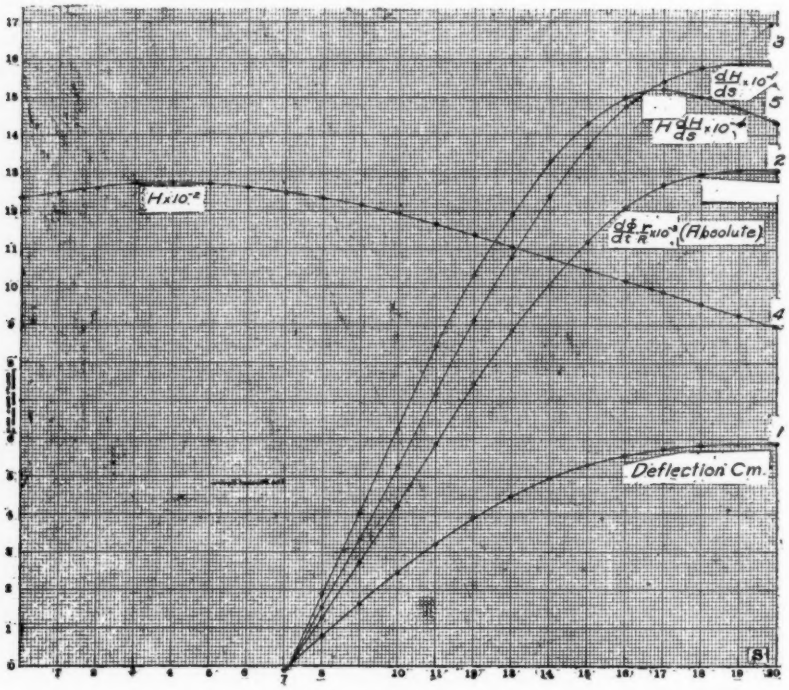
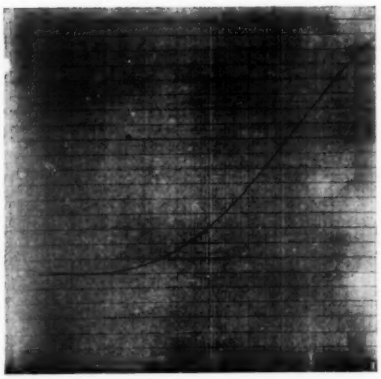


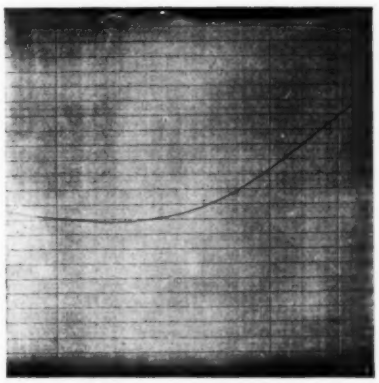
Fig. 3.

Curves showing the method of finding the maximum value of $H(dH/ds)$.

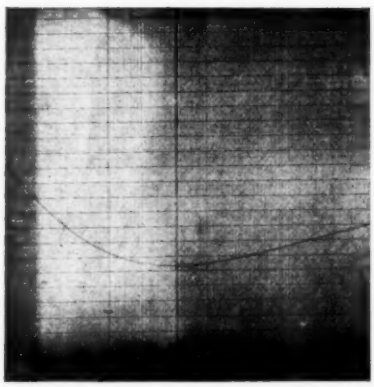
It is to be noticed in Fig. 3 that Curve 1, the deflection curve, and all the curves derived from this, do not cut the axis of abscissæ at the point where H is a maximum, although dH/dS is zero at this point. This displacement is due to lag in the galvanometer and takes place in the opposite direction when the motion of the magnetic field across the exploring-coil is reversed. This lag not only displaces the E.M.F. curve along the axis of abscissæ, but it distorts the curve as well. The lag in this curve, at any particular point, depends upon the rate at which the E.M.F. has been changing before this point is reached. As a result, the galvanometer curve given by a clockwise rotation of the magnet (see Fig. 4) will differ both in shape and position from the curve given by an anti-clockwise motion. Therefore, the error due to galvanometer lag cannot be overcome by shifting the curve so that it will cut the axis of abscissæ at the point where H is a maximum.



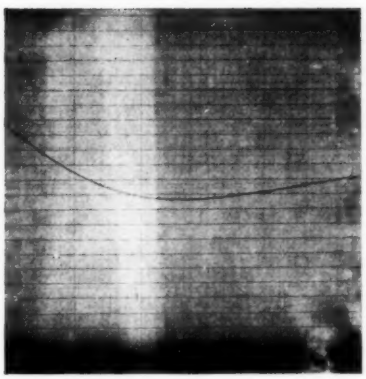
Curve 1.



Curve 2.



Curve 3.



Curve 4.

Fig. 4.

Specimens of the E.M.F. curves given by the Einthoven galvanonometer. Curves 1 and 2 were taken as the coil moved from stronger to weaker fields; Curves 3 and 4 refer to motion of the field in the opposite direction.

H. C. HAYES.

This difficulty was overcome as follows: Electromotive force curves were taken for various velocities of the magnetic field, and the maximum value of $H \cdot dH/dS$ determined for each velocity in the manner described. Fig. 3 shows that the value of $H \cdot dH/dS$ will be too small if the motion of the magnet is such as to carry the E.M.F. curve away from the point where H is a maximum and too large if the shift is in the other direction. It is also clear that the lag, and hence the resulting error, will become less as the velocity of the magnet is decreased, approaching zero as the velocity of the magnet approaches zero. If, therefore, the various values of $H \cdot dH/dS$ are plotted as ordinates against the corresponding velocity of the magnetic field as abscissæ, a curve passed through these points will, if extended, intercept the axis of ordinates at the true maximum value of $H \cdot dH/dS$.

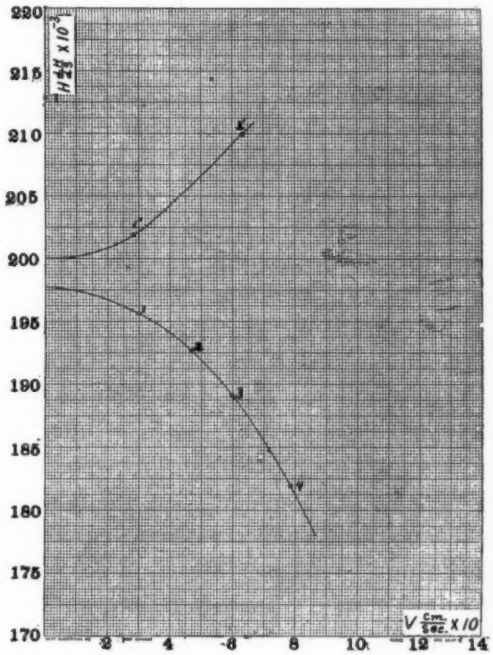


Fig. 5.

Curves showing the variation of $H\,dH/ds$, as determined by the second method, and V, the velocity of the field across the exploring-coil. The true value of $H(dH/ds)$ is given when the lag is zero, hence when V is zero.

The fact that this curve cuts the axis of ordinates perpendicularly aids greatly in the extrapolation. An excellent check on the work is furnished by determining such a curve for motion of the magnet in both clockwise and anticlockwise directions. These curves are shown in Fig. 5. The data for the lower curve was taken with the magnet rotating in a direction such that the E.M.F. curve was shifted away from the point where H is a maximum, and the upper curve refers to motion of the magnet in the opposite direction. The lines were drawn with the aid of a French curve. The intercepts give for the maximum value of $H \cdot dH/dS$ 1.98 $\times$ 10^5 and 2.00 $\times$ 10^5 respectively.

A third degree equation passed through points 1, 2, and 3, with the imposed condition that it cut the axis of ordinates perpendicularly, intercepts the axis of ordinates at 1.99 $\times$ 10^5, while a second degree

equation through points 1 and 2, and through points 1 and 3, with the same condition imposed, gives for the maximum values of $H \cdot dH/dS$ 1.99×10^5 and 1.98×10^5 respectively. A second degree equation passed through points 1′ and 2′ gives the value 1.98×10^5. The average value 1.985×10^5 is probably correct to within a small fraction of one per cent.

These values were obtained by using for the E.M.F. the drop across the terminals of the galvanometer, so they must be multiplied by the ratio R/r. The resistance of the whole circuit, R, was 2,854 ohms and the resistance of the galvanometer, r, was 2,784 ohms, giving for the ratio R/r the value 1.025, and this multiplied into 1.985×10^5 gives for the maximum value of $H \cdot dH/dS$ 2.03×10^5. The results obtained by the first method agree with this value to within 0.5 per cent.

The Specimen.—The specimen M consisted of 0.150 gr. of distilled water. This was contained in a small cylindrical capsule made of thin glass and drawn out at one end to a very fine capillary tube about 6 cm. long. By means of this tube the capsule could be filled and centered on the balance, as shown in Fig. 1. The deflection due to the action of the field on the water was taken as the difference between the deflection with the capsule empty and then filled.

The capsule was filled by first heating the glass and placing the end of the capillary in water. As the glass cooled some water was forced in to replace the air that had been expelled. This water was then driven out in the form of steam, carrying the remaining air with it. The open end was then placed under water while the steam was escaping, and as soon as the vapor condensed water entered and filled the tube. The capillary was then carefully heated and the water driven out in the form of steam. At the instant the flame was removed from the stem the capillary was sealed thus leaving a more or less perfect vacuum in the stem. The glass correction was not materially changed by this process.

The Force F_s was determined four separate times through the relation,

$$F = 1.882d,$$

where d is the deflection of the cross hair. The average value for d was -0.01175 cm., which gave for F_s the value -0.0221 dynes. The equation for X now becomes

$$-0.0221 = 0.150 \times 2.03 \times 10^5 \times X.$$

This gives for X the value -7.26×10^7 with the last figure doubtful.

Conclusion:—The value of the magnetic susceptibility of water at temperature 24° F. has been determined by Faraday's method, the difficulties of the method having been sufficiently overcome to make it

capable of giving accurate absolute results. The value of X as found by this method is in excellent agreement with the recent results obtained by Quincke's method, and seems to prove that this value is now known to within one per cent. Moreover, we may feel sure that this value is independent of the field, for the strength of the field used in this work was only 1,180 gausses while Sève, de Haas and Draper, and Weiss and Piccard used fields varying from 15,700 to over 21,500 gausses.

The excellent Einthoven galvanometer, which made possible the work presented in this paper, was obtained through a grant from The Rumford Fund.

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THE EFFECT OF MAGNETIZATION ON THE OPACITY OF IRON TO RÖNTGEN RAYS.

BY A. H. FORMAN.

IN a general way it would seem natural to expect that a change in the molecular arrangement of iron by magnetization, which produces the phenomenon of magneto-striction, should to some extent also affect the opacity of the iron to Röntgen rays. The investigations along this line have all given negative results. Experiments[1] in which the polarized secondary rays were used instead of the ordinary Röntgen rays have been tried with negative results.

In the winter of 1909, the author found indications of a positive character using a rough photographic method. The photographs indicated a slight increase in the opacity when the iron was magnetized in a direction parallel to the axis of the X-ray tube and perpendicular to the Röntgen rays. Because of their magnitude and the possibility that they were due to secondary causes, these results were not conclusive. It was therefore decided to see if they could be duplicated with an apparatus utilizing the ionizing power of the Röntgen rays. A method was devised which was much more sensitive than any used before and capable of detecting a change in the opacity of one part in ten thousand under the most favorable conditions.

Tests using this method, in which the iron was magnetized in a plane perpendicular to the Röntgen rays, with its direction of magnetization parallel to the axis of the X-ray tube, and also with its direction of magnetization perpendicular to this axis, gave negative results. The following is a description of the apparatus and the method of making the tests.

The apparatus finally adopted is shown in Fig. 1. The ionizing chambers consist of upright lead pipes (*CC*) soldered to a horizontal sheet of lead (*MM*). In the axis of each pipe is an insulated bare wire. The two wires are also connected to a grounding switch as shown. The ionizing chambers are both insulated and one maintained at a potential of + 220 volts and the other at a potential of − 220 volts. Two screens, one of lead and the other of a mixture of white-lead and putty, were used to stop rays other than those passing through the iron and entering the

[1] J. C. Chapman, Phil. Mag., Vol. 25, p. 792.

ionizing chambers. There is a possibility that very hard rays might even pass through these screens into the space below the charged plates and there ionize the air. This would cause a leakage to the wires and thus mask the effect of the charge which leaks to them in the ionizing chambers. To prevent this a grounded guard plate (G) was put below the lead sheets (MM). The potential of the wires is very small so that the leakage from the wires to the guard plate is quite small and negligible, while the ionization which results under the charged plates is taken care of by leakage to the grounded guard plate. If the Röntgen rays do get below the lead plates (MM) their effect is therefore inappreciable.

The piece of iron to be magnetized is fastened to the yoke of an electromagnet so that it completes the magnetic circuit as shown in Fig. 2. In the course of the experimental work, it was found that the stray field from the magnetizing coils disturbed the balance between the two chambers by deflecting the cathode ray in the X-ray tube. To prevent this two similar coils were placed under the magnetizing coils and connected in series with them so that they set up an opposing flux. This neutralized the effect of the stray field in the vicinity of the cathode ray. A screen of two sheets of iron was placed between the coils and the X-ray tube as an additional precaution. The magnetic effect was screened from the other piece of iron by iron laminations placed between the two pieces as shown in Fig. 1.

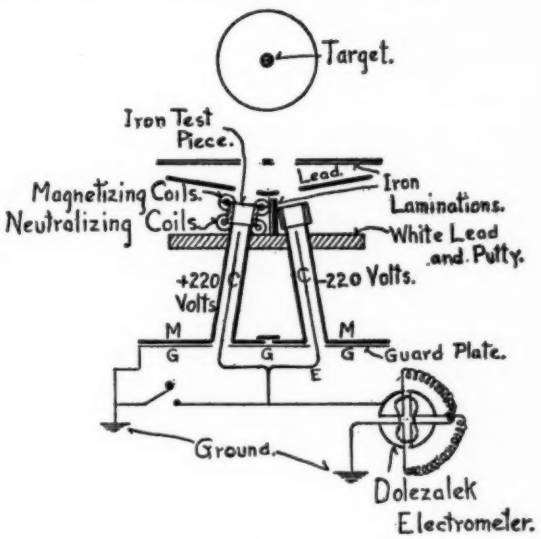

Fig. 1.

Perfect electrostatic screening of the electrometer system was obtained by use of a solid screening of tin-plate, except where the light was reflected from the electrometer mirror. This opening was screened by using a glass cell filled with a solution of NH_4Cl and grounded. The balance between the two ionizing chambers was made by sliding a narrow wedge of lead foil in and out across the top of one of the chambers, using a micrometer screw to adjust the position of the wedge. The elec-

trometer which was used gave a deflection of 84 cm. at a distance of 1 meter, with its needle charged to 62 volts, and a difference in potential of 1 volt across the quadrants.

With the two chambers charged oppositely as indicated, ionization in each tends to neutralize the charge that leaks to the wire from the other, and when there is a perfect balance no charge accumulates on the wire. A very slight change in ionization of either chamber causes a piling up of a charge on the wire, and the deflection of the electrometer is proportional to the change in ionization and the time that the grounding switch is open.

The sensibility of this system is so great that difficulty was experienced at first in getting a balance between the chambers because of disturbances due to a change in the hardness of the X-ray tube. A very slight change in the potential difference across the X-ray tube would destroy the balance. It was therefore necessary to connect a high tension electrostatic voltmeter across the X-ray tube, and take observations for definite potential differences. In taking the readings the following method was used. With the grounding switch closed, a current was sent through the X-ray tube and as soon as this tube had reached the desired condition, the grounding switch was opened. After the switch had been open for five seconds the current through the X-ray tube was stopped and the electrometer deflection observed. The deflection thus obtained is proportional to the charge which accumulates in five seconds, due to the difference in ionization in the chambers.

The effect of the magnetization of the iron on the change of the balance was tried with rays of three degrees of penetrability. Measured by the equivalent spark between sharp points, the potential difference across the X-ray tube was respectively 4.2, 7 and 10.5 cm. The iron used was 0.08 mm. thick and had the following chemical analysis: 0.093 per cent. silicon, 0.06 per cent. sulphur, 0.008 per cent. phosphorus, 0.24 per cent. manganese, 0.1 per cent. carbon and 99.47 per cent iron.

It was magnetized to saturation by sending a large current through the magnetizing coils and it was demagnetized by sending 60-cycle alternating current through the coils and reducing the current by steps to zero.

Tests were made in which the iron was turned with its direction of magnetization parallel to the axis of the X-ray tube, and also with its direction of magnetization perpendicular to this axis. Since the tube varied in hardness even during an observation, it was necessary to take a number of readings with the iron first magnetized and then demagnetized. To guard against any shift due to secondary causes, observations were made in pairs, first with the iron demagnetized and then with the iron magnetized.

The sensibility of the apparatus was tested by finding the smallest change in the area of the iron, penetrated by the rays entering the chamber, which produced a change of balance appreciably greater than any effect noticed in the magnetization tests. The area of the iron penetrated by the rays entering the ionizing chamber was slightly less than the area of the ionizing chamber opening, since the iron was a little distance above the chamber (see Fig. 2). The diameter of the effective area was found experimentally by sliding a lead strip (edge on) across the top of the iron and parallel to a diameter of the opening of the chamber, and finding the first point where a change of balance was produced. It was then moved across the entire opening and another point found where a change of balance was produced. The distance between these points is the diameter of the circle penetrated by the rays entering the ionizing chamber. To get the smallest change in area which could be detected, the lead strip which extended about one fourth of the diameter inside the circle, was moved through the smallest distance that would appreciably disturb the balance. This distance multiplied by the thickness of the lead strip gives the increment in area. The total area is found by subtracting the area covered by the lead strip from the area of the effective circle, and the sensibility is expressed as the ratio of the change in area to the total area. In one of the tests the values were as follows: Diameter of effective circle 19 mm. Thickness of strip 0.86 mm. Area covered by the strip 3.44 sq. mm. Distance strip was moved to change balance 0.05 mm. Hence the sensibility was 0.043 divided by 280, or 0.00015.

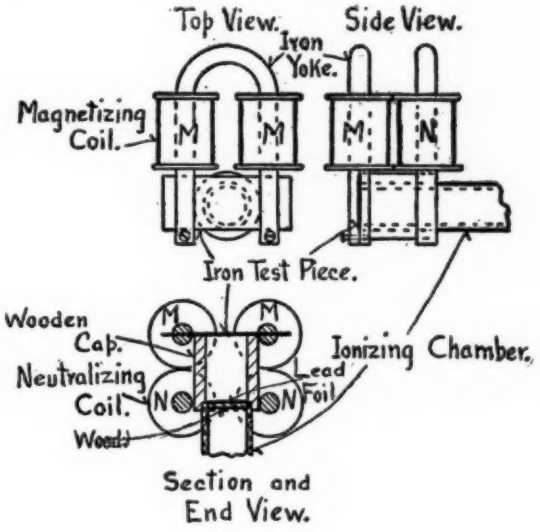

Fig. 2.

The sensibility depends upon the total number of rays passing through the iron into the ionizing chamber, and this depends on the degree of penetrability of the rays and the current passing through the X-ray tube. The sensibility was therefore determined for each degree of penetra-

TABLE I.

Magnetic Field Perpendicular to Axis of X-ray Tube.				Magnetic Field Parallel to Axis of X-ray Tube.				Sensibility Test. Proportional Change of Area=0.0015.			
X-ray Tube.		Electrometer Deflect.		X-ray Tube.		Electrometer Deflect.		X-ray Tube.		Electrometer Deflection.	
Miliamps.	Equivalent Spark-gap.	Iron Demagnetized.	Iron Magnetized.	Miliamps.	Equivalent Spark-gap.	Iron Demagnetized.	Iron Magnetized.	Miliamps.	Equivalent Spark-gap.	Zero Setting of Lead Strip.	Setting Changed 0.5 mm.
5	4.2 cm.	57		5	4.2 cm.	64.5		5	4.21 cm.	64	
5	4.2	71.5		5	4.2	61.5		5	4.21	64	
5	4.2		49.5	5	4.21		64.5	5	4.2	69.5	
5	4.23		48	5	4.2	63		5	4.22	58.5	
5	4.23	57.5		5	4.21	66		5	4.23		43
5	4.2	51.5		5	4.22		62	5	4.23		42.5
5	4.25		39.5	5	4.23		70	5	4.23	52.5	
5	4.24		50	5	4.23	66		5	4.25	52.5	
5	4.2	25						5	4.25		32
5	4.25	40						5	4.26		21
5	4.25		46					5	4.28	27.5	
5	4.25		41.5					5	4.25	28.5	
5	4.25	35.5						5	4.26	24.5	
5	4.25	24						5	4.27		−5
								5	4.26		−7.5
Mean deflection Difference = 0.45		45.25	45.7	Mean deflection Difference = 1.3		64.2	65.5	Mean deflection Difference = 27.9		48.9	21

TABLE II.

Magnetic Field Perpendicular to Axis of X-ray Tube.				Magnetic Field Parallel to Axis of X-ray Tube.				Sensibility Test. Proportional Change of Area = 0.00015.			
X-ray Tube.		Electrometer Deflect.		X-ray Tube.		Electrometer Deflect.		X-ray Tube.		Electrometer Deflect.	
Miliamps.	Equivalent Spark-gap.	Iron Demagnetized.	Iron Magnetized.	Miliamps.	Equivalent Spark-gap.	Iron Demagnetized	Iron Magnetized.	Miliamps.	Equivalent Spark-gap.	Zero Setting of Lead Strip.	Setting Changed 0.05 mm.
3	7.3 cm.	55.5		4	6.95 cm.	37.5		4.7	7 cm.	−55	
3.2	7.1	−65		4	6.8	−21.5			7.07	−66	
3.2	7.1		−180	4	6.9		19.5	5	7	−68	
3.15	7.2		−180	4	6.95		180	5	7		170
3.3	7.35	89		4	6.95	105		5	7.03		140
3.5	7.1	46		4.2	6.75	−12		4.5	6.85		90
3.6	7.2		−54	4.2	6.75		−31	4.8	6.95	−180	
3.7	7.25		−42	4.1	6.8		−14.5				
4	7.1	−91.5		4	6.8	56.5					
4	7.1		35.5	4.1	6.75	21					
3.9	7.1		40	4.15	6.75		21				
3.9	7.1	−180		4.15	6.75		−31				
3.6	7.1		−33.5								
3.5	7.1		−110								
3.5	7.1	−180									
3.7	7.1	−108									
Mean deflection. Difference = 11.		−54.5	−65.5	Mean deflection. Difference = 8.1		32.1	24	Mean deflection. Difference = 225.		−92	133

TABLE III.

Magnetic Field Perpendicular to Axis of X-ray Tube.				Magnetic Field Parallel to Axis of X-ray Tube.				Sensibility Test. Proportional Change of Area = 0.00018.			
X-ray Tube.		Electrometer Deflect.		X-ray Tube.		Electrometer Deflect.		X-ray Tube.		Electrometer Deflect.	
Miliamps.	Equivalent Spark-gap.	Iron Demagnetized.	Iron Magnetized.	Miliamps.	Equivalent Spark-gap.	Iron Demagnetized.	Iron Magnetized.	Miliamps.	Equivalent Spark-gap.	Zero Setting of Lead Strip.	Setting Changed 0.06 mm.
1.5	10.55 cm.	59		1.9	10.5 cm.	23		—	9.65 cm.	77	
1.6	10.5	35		1.9	10.6	88.5		—	9.55	80	
1.55	10.53	26		1.95	10.6	84.5		1.5	9.45	2.1	
1.85	10.53		20	1.8	10.6	100		1.5	9.45		−23
1.85	10.53		22	1.8	10.6	180		1.4	9.55		19
1.85	10.53		65	—	10.55		17	1.5	9.45		−27
1.8	10.5	−2		1.9	10.6		170	1.5	9.35		23
1.8	10.42	−2.5		2	10.6		18	1.2	9.35		−180
1.7	10.5	27.5		1.9	10.45	−180		1.4	9.4		−175
1.7	10.53	37		1.95	10.6	89.5					
1.7	10.53		31	1.9	10.55		75				
1.5	10.53		42.5	1.95	10.5		30				
1.6	10.53	38.5		1.95	10.45	−170					
1.5	10.53	54.5		1.8	10.55	33					
1.5	10.62		77.5	1.8	10.6		87				
1.65	10.55		24	1.85	10.6		64				
1.8	10.5		33	1.8	10.55	26					
1.55	10.53	52		1.8	10.55	18					
1.5	10.5	50		1.85	10.5		−31.5				
1.5	10.53		63	1.9	10.45		−180				
1.5	10.53		53	1.85	10.5	−47					
1.5	10.53	57									
Mean deflection. Difference = 1.1		38.3	37.2	Mean deflection. Difference = 8.6		20.4	31	Mean deflection. Difference = 120.1		59.6	−60.5

bility. For the softer rays the sensibility was 0.0015, for the medium rays it was 0.00015 and for the hardest rays it was 0.00018. The reason that the sensibility for the run with the hard rays was less than that of the medium rays is that the X-ray tube used for the hard rays was smaller and a smaller current was sent through it.

The preceding tables give the results of the tests.

In some of the tests the values differ considerably. This is because of the high sensibility of the apparatus and the fact that the condition of the X-ray tube varied. It will be noticed that in the magnetization tests the mean values agree closely, thus showing that as a whole there is no change in the balance. In the sensibility tests the change in ionization is evident from the mean values of the deflections and their differences. We therefore conclude that there is no effect due to the magnetization of the iron, with the magnetization in a plane perpendicular to the Röntgen rays and either parallel or perpendicular to the axis of the X-ray tube, unless it is less than the effect produced by shifting the lead strip. In the case of the soft rays it must be less than 0.0015, medium rays less than 0.00015, and hard rays less than 0.00018. In the earlier tests an effect was observed when the magnetizing current was on, but this was found to be due to the deflection of the cathode ray of the X-ray tube, which is probably the cause of the positive results obtained at first photographically. Investigations have been started using a field of 3,500 gauss parallel to the Röntgen rays and perpendicular to the plane of the iron.

As is evident, the method which has been used in this work has the great advantage that even though the ionization in each chamber is quite large, the electrometer measures only the small difference in the ionization. And since this method gave such a high degree of sensibility, it seems to the author that it would be quite useful in other fields of investigation where the changes sought for are likely to be quite small.

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PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE SEVENTY-FIRST MEETING.

A REGULAR meeting of the Physical Society was held in Fayerweather Hall of Columbia University on Saturday, February 28, 1914.

The following papers were presented:

A Method of Rapidly Extracting, Purifying and Compressing Radium Emanation. WILLIAM DUANE.

On the Density of Radiant Action. WILLIAM DUANE.

Secondary Electron Emission from a Hot Cathode Caused by Positive Ion Bombardment. IRVING LANGMUIR.

Additional Data on the Illumination-Photoelectric Relation. H. E. IVES.

An Improved Mercury Pump. A. H. PFUND. (By title.)

Note on the Photoelectric Cell as an Analyzer for Polarized Light. By E. MERRITT.

A Thermoelectric Method of Determining the Purity of Platinum Ware. G. K. BURGESS and P. D. SALE.

The Electric Resistance and Critical Ranges of Pure Iron. G. K. BURGESS and I. W. KELLBERG.

Radiation Constants of a Nitrogen-Filled Tungsten Lamp. W. W. COBLENTZ. (By title.)

The Villari Critical Point in Ferromagnetic Substances. S. R. WILLIAMS. (By title.)

Motion of a Radiating Oscillator. E. B. WILSON. (By title.)

On the Asymmetric Distribution of Velocities of Photo-Electrons from Platinum Cathode Films. OTTO STUHLMANN, JR. (By title.)

A. D. COLE,
Secretary.

AN IMPROVED MERCURY LAMP.[1]

BY A. H. PFUND.

SEVERAL years ago[2] the writer described a convenient type of mercury lamp which served as a source of visible as well as ultra-violet radiations of mercury vapor. Much experience with this lamp has shown that, while it

[1] Abstract of a paper presented at New York meeting of Physical Society, February 28, 1914.

[2] Astrophysical Jnl., 27, p. 299, 1908; Ztschr. Für. Wiss. Phot., 4, p. 329, 1908.

has its good features, it also possesses the defect that the iron electrode used gives rise to a yellow coating on the glass—thus limiting the life and usefulness of the lamp to an unnecessarily short period. Recently the lamp has been redesigned and the objectionable features have been eliminated while the good ones have been retained.

The process of making the lamp is as follows: a glass tube 2.5 cm. in diameter is drawn down at one end to a diameter of 1 cm. and is closed; a bulb is then blown and, while the glass is still soft, the lower end of the tube is pushed up, thus creating the annular trough shown in the dotted lines of Fig. 1. Next a side tube *B*, Fig. 2, is attached; a small bulb is blown in the tube *A* and the platinum electrodes are sealed in. The inner end of the wire in *A* is preferably bent as indicated to avoid any possible arcing between this wire and the mercury when the lamp is started subsequently by tilting. The lamp as such is finished after the side tube *C* has been introduced and a lip (ground flat with carborundum powder) has been turned on the upper end of the tube. After the lamp has been cleaned and dried thoroughly, distilled mercury is introduced to the indicated amount and a quartz plate is attached to the open end of the tube by means of sealing-wax.

Fig. 1.

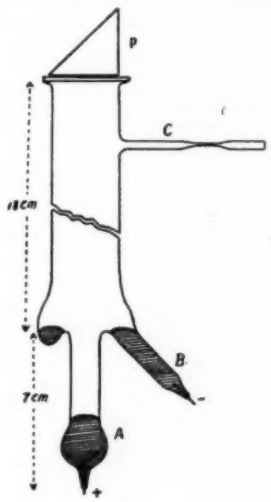

Fig. 2.

After having drawn down the tube *C*, the lamp is attached to a mercury pump and the usual process of exhaustion and boiling of mercury is carried out. The lamp is then connected to a source of direct current (110 volts) through an auxiliary resistance of 5–6 incandescent lamps in parallel, thus reducing the current to about 2 amperes. The discharge is started, either by a high-potential discharge or by boiling the mercury in tube *A* so violently that it rushes up to short-circuit the lamp for an instant. While the lamp is still on the pump, the discharge is continued for an hour or more to drive off gases and vapors. After sealing off from the pump, the lamp is ready for use.

Whenever it is desired to start the lamp subsequently, the tubes *A* and *B* are heated cautiously by means of a bunsen burner and the lamp is tilted so that the mercury from *A* flows *directly* into the top of the tube *B*. Upon returning the lamp to its vertical position, a brilliant discharge sets in. By connecting the lower electrode *A* to the positive end of the source of current, the discharge in the tube *A* becomes very steady, while a brilliant spot dances

about on the surface of the mercury in the annular trough. This annoying feature is eliminated by covering the outside of the trough with black paint.

The mercury which condenses in the upper part of the lamp gradually runs down into the trough which overflows periodically into the tube *A*. By constricting, suitably, the larger tube above the trough, it is possible to cause a portion of the falling mercury droplets to reach *A* directly—thus eliminating the sudden overflow previously mentioned. In the great majority of cases, however, the lamp as described, will meet all requirements. In order to avoid the condensation of mercury on the quartz window, the lamp is sealed off from the pump at a pressure of about 1 mm. (thus avoiding the luminous flame which shoots up the tube at high vacua). If, in spite of these precautions droplets persist in forming on the window, this difficulty is overcome by wrapping strips of wet cloth about the upper part of the lamp. If it be desired to utilize the ultra-violet radiations, a totally reflecting quartz prism is introduced at *P*.

While lamps of the type described have been tested for only several months, no defects of design have become apparent.

JOHNS HOPKINS UNIVERSITY,
February, 1914.

CONTRIBUTIONS TO THE THERMODYNAMICS OF SATURATED VAPORS.[1]

BY J. E. SIEBEL.

THE author finds that the expansion line in indicator diagrams of first class Corliss engines follows an isothermal curve, an outcome which he shows is also theoretically quite in harmony with Gibb's phase rule and which permits the correct calculation of the theoretical work of such engines after the familiar formula of isothermal expansion. The results of these calculations, it further appears, when adapted to the concepts of a theoretically perfect reversible cycle, are amenable to the compound rate of conversion expressive of the second law given by Clausius as

$$W = \frac{Q(t - to)}{T} + \frac{Q_1(t - to)}{T},$$

in which formula W represents the maximal theoretical work obtainable in an ideal reversible steam engine cycle, between the temperatures t and to when Q signifies the initial heat of vaporization at t degrees and Q_1 the heat becoming free in the expansion stage by the condensation of water; T representing the absolute temperature of the boiler equal to t degrees common temperature.

[1] Abstract of a paper presented at the Atlanta meeting of the Physical Society, December 31, 1913.